Inflation and Real Wage Inequality: Empirical Evidence and a Model of Frictional Markets

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November 22, 2012

Abstract

According to US data over the period from 1994 to 2008, inflation has a positive effect on residual real wage inequality, and this effect primarily operates through a stronger negative effect on low wages relative to high wages. To explain this effect, we introduce random matching and wage posting into the framework by Berensen, Menzio and Wright (2011). In the model, uncoordinated job searches by workers give rise to an equilibrium in which firms are matched with zero, one or multiple job applicants. This mechanism generates wage dispersion among identical workers. Inflation influences the wage distribution directly through its influence on the real profits of firms and indirectly through a spillover effect. Quantitatively, the calibrated model can explain approximately two-thirds of the observed adjustment of wage dispersion in response to changes in inflation.

JEL Categories: E40, J30
Keywords: Inflation; Wage Inequality; Search; Matching

1 Introduction

The influence of inflation on wage inequality is one of the most extensively researched topics in monetary economics and labor economics. Although many studies have focused on the relationship between inflation and real wage inequality (for example, Hammermesh, 1986; Erikson and Ichino, 1995; Bulir, 2001), the effect of inflation on the residual real wage

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inequality has received little attention in the literature. In this paper, we develop a search-theoretic general equilibrium model to study the effect of inflation on real wage inequality among identical workers, which is called residual wage inequality in the labor literature.

The empirical motivation for this study is as follows: existing empirical work that examine the relationship between inflation and residual wage inequality are sparse and outdated. In this paper, we provide the most recent empirical evidence for the relationship between these two variables. We use the Current Population Survey (CPS) data at quarterly frequency for the period from 1994 to 2008. Since 1994, the CPS provides worker-level information on a variety of demographic characteristics and features of jobs, which enables us to measure residual wage inequality as precisely as possible. Following the labor literature, we construct residual wage distribution after controlling for the observed differences in workers and jobs and then calculate our measure of residual real wage inequality. In our benchmark case, residual wage inequality is defined as the ratio of high wages (the 75th percentile of the wage distribution) to low wages (the 25th percentile of the wage distribution). Our constructed residual wage inequality is approximately 1.54, which is fairly close to various measures in recent studies. For example, Hornstein et al (2011) reports mean - min ratios between 1.5 and 2 based on the 2000 OES survey and the 1967-1996 waves of the PDIS survey. In terms of its relationship with inflation, we find that residual real wage inequality is positively correlated with inflation after controlling for labor productivity. The data suggests that when inflation is doubled, the wage inequality increases by 1.23 percent. This result remains robust when different measures of wage inequality are used. In addition, when we examine the influence of inflation on different wage levels, we find that a change in inflation negatively affects low wages more than high wages, and this result suggests that a change in real wage inequality is primarily driven by a decrease in the low wage.

Theoretically, this paper contributes to the literature in that it develops a search-theoretic general equilibrium monetary framework to address two important issues: the reason that

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1 We chose to use data through September of 2008 because the global financial crisis that peaked in late 2008 and early 2009 resulted in a significant decline in economic activity and prolonged unemployment.
wage inequality exists among identical workers and the manner in which a change in inflation affects residual wage inequality. In studying the first issue, the existing literature either relies on on-the-job searches, such as Kumar (2008), or uses a two-tier unemployment insurance system, such as Albrecht and Vroman (2001), to generate wage inequality among homogenous workers. In this paper we adopt a different approach: wage inequality results from uncoordinated job searches in a frictional labor market. Specifically, we incorporate uncoordinated job searches into the model of Berentsen, Menzio and Wright (2011; henceforth, BMW). The labor market in our model features a variation of the large economy with random matching and wage posting as presented in Julien, Kennes and King (2000). We assume that firms are capacity constrained in the sense that each firm has only one vacancy and that this vacancy can be offered to a maximum of one unemployed worker. Firms that are seeking to hire workers post a menu of wages that they are willing to pay to attract applicants. Workers subsequently choose to apply to one and only one firm while being ignorant of the decision of other workers. The uncoordinated nature of the job-search process implies that some firms attract multiple applicants, whereas others do not. This mechanism generates wage inequality among identical workers. Specifically, a menu posted by a searching firm consists of two wage levels: a high wage and a low wage. When only one worker approaches a searching firm, we assume that this worker holds all of the wage determination power and extracts all of the gains from a match with a high wage. Thus, the high wage is the wage at which the firm is indifferent between employing the worker and not employing the worker (and thus being inactive). By contrast, if more than one worker approaches a firm, then only one of the applying workers successfully forms an employment match. Competition between applicants for the sole position results in the firm extracting all gains from the match by offering a low wage. The low wage is the wage at which the successful applicant is indifferent between working and not working. In equilibrium, the low wage is a weighted average of the high wage and the value of non-market activities encountered by an unemployed worker (primarily unemployment insurance (UI) benefits in our model).
For the second issue, we conclude that changes in inflation positively affect wage inequality among identical workers, as observed in the data. The central insight of the mechanism that delivers this positive relationship lies on a spillover effect. In the context of high inflation, the real profits of firms decrease, which directly reduces the high wage (high wage is equal to real profits) and thus, decreases the low wage. In addition, the decrease in real profits in the goods market reduces the incentives of firms to post vacancies in the labor market, which translates into a tighter labor market condition and makes workers less likely to locate a high-wage job. We refer to this effect as a spillover effect (a spillover from the goods market to the labor market). The spillover effect moves the low wages closer to the amount of UI benefits, which further reduces the low wages.

We then calibrate the model to the US data and quantify the steady-state effect of inflation on wage inequality and on wage levels. In the baseline calibration, our model can account for 80 percent of the observed moments in low wages, and the predicted reaction of high wage is also fairly close to its empirical counterpart. When we deviate from the baseline case by lowering UI benefits by increasing the mark-up ratio and convexity of the cost function, our model can generate a 0.76 percent increase in wage inequality when inflation is doubled; this increase, is approximately two-thirds of the observed adjustments in wage inequality (1.23 percent) in the data.

Our paper is related to the empirical literature on the relationship between inflation and wage inequality cited earlier, and we note that most of these studies explore the relationship for the period prior to 1990. Generally, this empirical literature has reported a negative relationship between inflation and wage inequality, which differs from our results and contradicts the findings of the theoretical literature. In the theoretical literature, some existing works (e.g., Sheshinski and Weiss (1977), Benabou (1988, 1992), and Diamond (1993)) as-

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2Hamermesh (1986) analyzes the relationship above by using 20 two-digit manufacturing industries for the 1955-1981 period in the United States. He finds that greater inflation reduces the dispersion in relative wage changes. Similar findings are reported in Italy by Erikson and Ichino (1995), who study metal-manufacturing firms for the 1976-1990 period. However, Bulir (2001) argues for a positive relationship between these two variables using cross-country data.
sume that because of the nominal rigidity, firms follow a \([s; S]\) strategy in wage setting. The aforementioned authors propose that an increase in the rate of inflation increases wage dispersion. However, the existence of rigidities is responsible for these findings. The mechanism at work in our model is much richer and subtler: an increase in inflation reduces the real profits of firms, and such decreased profits directly lower both high wages and low wages. In addition, a lower real profit causes firms to post fewer vacancies in the labor market. A tighter labor market reduces the probability of a worker finding a high-wage job. This decreased probability incurs further downward pressure on the low wage by moving this wage in the direction of UI benefit amounts. As a result, wage inequality increases. An additional problem with existing models is that they lack a microfoundation for money. Such models cannot address many questions, such as the question as to why money acts as a medium of exchange. In this paper, we explicitly model the frictions that render money "essential" (i.e., higher social welfare can be achieved through the use of money than without it.)

The paper that is most closely related to our work is the research of Kumar (2008), who also explores the connection between inflation and residual wage inequality in a search model. However, our paper differs from his work in several ways. First, Kumar (2008) relies on on-the-job searches to generate wage inequality among identical workers, whereas in our model, such wage inequality results from an uncoordinated job search in a large labor market. Second, our paper examines inflation and residual wage inequality information, whereas Kumar (2008), and the majority of the existing literature (for example, Hammer-mesh, 1986), considers only the relationship between wage inequality and inflation. The distinction between wage inequality and residual wage inequality is crucial in a model with homogenous workers, which is the case in Kumar (2008) and in our model. In this sense, we argue that residual wage inequality is a more appropriate measure of wage inequality among identical workers. Third, Kumar (2008) does not study the model’s empirical performance; in contrast, we conduct a quantitative analysis to examine the model fit.

The remainder of the paper is organized as follows. Section 2 documents the key facts
characterizing the relationship among wages, residual wage inequality and inflation for the period from 1994 to 2008. Section 3 develops a model based on BMW but with uncoordinated searches in the labor market. Equilibrium is established, and key properties governing the relationship between wages and inflation are discussed. Section 4 calibrates the model to the data in the US. Section 5 concludes the paper.

2 Data and Facts

The data source that we use is the Current Population Survey (CPS). Beginning with 1994, CPS contains a variety of demographic characteristics, including age, sex, race, marital status, and educational attainment. Moreover, these data contain interesting information on jobs, such as occupation and industry information. This information enables us to calculate precise measures of residual wage inequality. We choose a quarter as a period and examine all periods from 1994 to 2008. Appendix A provides a description of the survey and of the selection criteria that we adopt. With respect of the measure of wages, we choose hourly wage rather than earnings to account for the possibility that differences in earnings may reflect difference in hours worked.

To obtain the relationship among wages (low and high), wage inequality and inflation, we analyze the data as follows. First, we eliminate all wage variations caused by observed differences in workers and jobs. Following Hornstein et al (2011), for every quarter in the 1994-2008 period, we run an OLS regression on the cross-section of the nominal hourly wage for an individual’s main job (in log), and we control for gender, race, marital status, education, experience (age minus years of education minus five), industry, occupation, a

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3 The CPS data that are used in this work date back to 1989. The reason for excluding data before 1994 is that the data on hourly wages (main jobs) that are used to construct the residual wage dispersion are not available for the 1989-1993 period. The data on wages that are available during this period refer to hourly earnings, which differs from hourly wages in several respects. First, hourly earnings contain hourly wages, tips, commissions and other types of compensation. Second, the question regarding hourly earnings is answered by any member of a family, whereas hourly wages measure wage rates received by a family head. Third, hourly earnings reflect earnings in all jobs, whereas hourly wages only reflect information from one’s primary job. Because of these differences, we exclude the data before 1994.
dummy for union members, and the interaction between occupation and experience to capture occupation-specific tenure profiles. This procedure provides us a time series of residual nominal hourly wage. Second, we divide the residual nominal wage by the consumer price index. The resulting series is the residual real wage (in log). Finally, we use an exponent with a base of e to "undo" the logarithm. We convert the residual real wage (in log) back to its standard value because this method ensures positive wage inequality as defined below.

We now calculate our indices of wage inequality. In the benchmark case, we define high wage and low wage as, respectively, the 75th ($wage_{75}$) and 25th percentiles ($wage_{25}$) of the residual real wage in quarter $t = 1, ..., T$. Our measure of wage inequality ($WI_{75}$) in quarter $t$ is given by

$$ WI_{75t} = \frac{wage_{75t}}{wage_{25t}}. $$

Figure 1 shows scatter plots between $WI_{75}$ and quarterly CPI inflation. It is evident that there exists a positive relationship between these two variables.

Because low wages are likely to be subject to outliers, as a check of robustness, we also consider four other measures of wage inequality, namely, $WI_{80}$, $WI_{85}$, $WI_{90}$ and $WI_{95}$, which are equal to the high wage (80%, 85%, 90% and 95% percentiles of the residual wage
distribution, respectively) divided by the corresponding low wage (20%, 15%, 10% and 5% percentiles of the residual wage distribution, respectively). The results show that the positive relationship between inflation and wage inequality is robust across all of these four cases.

Table 1

<table>
<thead>
<tr>
<th>Wage Inequality (Residual)</th>
<th>WI75</th>
<th>WI80</th>
<th>WI85</th>
<th>WI90</th>
<th>WI95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.54</td>
<td>1.72</td>
<td>1.97</td>
<td>2.35</td>
<td>3.12</td>
</tr>
</tbody>
</table>

Table 1 summarizes the calculated values of residual real wage inequality in all cases. The baseline result of 1.54 is fairly close to the estimates in recent studies. For example, Hornstein et al (2011) find mean - min ratios between 1.5 and 2 based on the November 2000 OES survey and the 1967-1996 waves of the PDIS survey. Acemoglu (2002), Lemieux (2006) and Autor et al (2008) find 50 – 10 wage percentile ratios between 1.7 and 1.9 for male workers.

In the second step, we more closely examine this relationship, especially the effects of inflation on high wage and low wage. We run three regressions separately for each quarter. Each regression has the following general form \( \log(Y_t) = \beta_0 + \beta_1 \log(\text{inflation}_t) + \beta_2 \log(\text{productivity}_t) \), where \( Y_t \) is the high wage, low wage and wage inequality. Productivity refers to labor productivity, which is measured by output per worker in the business sector. \(^4\) \( \beta_1 \) is the coefficient of interest, and it measures the elasticity of high wages, low wages, and wage inequality with respect to inflation. Table 2 reports the regression results of the benchmark case and the four other measures of wage inequality.

\(^4\)We include productivity in the regression because it has been widely agreed that productivity affects high wages to a large extent and thus ultimately influences wage dispersion. Omitting productivity in the regression would cause an endogeneity problem.
Table 2
Effects of Inflation on Wages and Wage Inequality

<table>
<thead>
<tr>
<th>Variables</th>
<th>High Wage</th>
<th>Low Wage</th>
<th>Wage Inequality (WI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log_(wage75)</td>
<td>log_(wage80)</td>
<td>log_(wage85)</td>
<td>log_(wage90)</td>
</tr>
<tr>
<td>log(inflation)</td>
<td>-0.0261*** (0.0084)</td>
<td>-0.0266*** (0.0086)</td>
<td>-0.0260*** (0.0092)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9670</td>
<td>0.9652</td>
<td>0.9589</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Low Wage</th>
<th>Wage Inequality (WI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log_(wage25)</td>
<td>log_(wage20)</td>
<td>log_(wage15)</td>
</tr>
<tr>
<td>log(inflation)</td>
<td>-0.0383*** (0.0086)</td>
<td>-0.0411*** (0.0086)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9686</td>
<td>0.9694</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Wage Inequality (WI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log_(WI75)</td>
<td>log_(WI80)</td>
</tr>
<tr>
<td>log(inflation)</td>
<td>0.0123** (0.0048)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.2210</td>
</tr>
</tbody>
</table>

Notes: 1.Standard errors in parentheses. 2.***p<0.01; **p<0.05.

Two findings are prominent here: first, the positive relationship between inflation and wage inequality is statistically significant across all five cases. In the benchmark case, the elasticity of $WI_{75}$ with respect to inflation is approximately 0.0123, which suggests that if the inflation rate doubles (e.g., from 2 to 4 percent), then the wage inequality will increase by 1.23 percent. Second, although inflation affects both high wages and low wages negatively, it is clear that the negative effect is much stronger on low wages than on high wages. The benchmark case shows that a low wage ($wage_{25}$) will decrease by 0.38 percent as a result of a 10 percent increase in inflation, whereas a high wage will decrease only by 0.26 percent. This result suggests that the increased wage inequality in response to an inflation rise is primarily driven by declines in the low wage.
3 The Model

We now develop a model of frictional markets in which higher anticipated inflation rates cause an increase in wage inequality. The model is based on the search-theoretic general equilibrium monetary model in BMW.

Time is discrete, and agents are infinitely lived. Each time period is characterized by three distinct markets in which agents meet to conduct economic activity: a decentralized labor market, a decentralized goods market and a centralized goods market. The decentralized labor market is designed in the spirit of the labor search theory of Mortensen and Pissarides (1994) and is referred to as the MP market. The decentralized goods market is designed in the spirit of the monetary search theory of Lagos and Wright (2005) and is referred to as the LW market. The centralized goods market functions as a clearing market and is referred to as the Arrow-Debreu or AD market. We assume the markets open sequentially in each period, beginning with the MP market and followed, in turn, by the LW and AD markets, as shown in Figure 2. All agents discount each time period at a uniform rate $\beta \in (0, 1)$. This implies that discounting occurs following AD market activity and precedes MP market activity.

![Figure 2 Timing]

There are two types of private agents: firms ($f$) and workers ($h$). The set of potential firms is arbitrarily large, and the set of workers is normalized to $[0, 1]$. Although all workers are continuously active in the model, there always exist some inactive firms. Inactive firms may choose to become active at any time. Such firms enter the MP market when it is possible
to make positive economic profits. Workers provide labor and consume output to maximize lifetime utility. Firms employ labor and sell their output so as to maximize profits, which are then paid as dividends to workers.

Money is intrinsically useless, perfectly divisible and storable. The quantity of fiat money per capita at the beginning of period $t$ is $M_t$. We assume that $M_{t+1} = (1 + \pi)M_t$, where $\pi$ is constant and new money is injected by lump-sum transfers to workers in the AD market. The price of goods in terms of money in the AD is $p_t$. We restrict our attention to steady-state equilibria in which the real value of aggregate money balances $M/p$ is constant. This implies $p_{t+1} = (1 + \pi)p_t$, and $\pi$ is the inflation rate between this and the next AD market.

To facilitate the presentation of the model, we omit time indices and use $\hat{a}$ to denote the value of any variable $a$ in the next period. In addition, we convert all nominal variables into real terms in the following manner. When an agent enters the AD market with $m$ dollars, we let $z = m/p$ denote his real balances. He then brings $\hat{z} = \hat{m}/p$ out of this market and into the next period’s MP market, still deflated by $p$. If he is not matched in the next LW market, then he carries $\hat{z}$ into the next AD market. Its real value is then given by $\hat{z}\hat{\rho}$, where $\hat{\rho} = p/\hat{p} = 1/(1 + \pi)$ converts $\hat{z}$ into the units of the numeraire in that market.

The value functions for the MP, LW and AD markets are $U^j_e$, $V^j_e$ and $W^j_e$, respectively. The value functions depend on agent type $j \in \{h, f\}$ and employment status $e \in \{0, 1, 2\}$, where $e = 2$ indicates that the agent is currently in a high-wage match in the MP market, $e = 1$ indicates that the agent is currently in a low-wage match in the MP market, and $e = 0$ indicates the agent is unmatched in the MP market. Value functions also depend on other state variables that are defined below. In the following discussion, we consider a representative period $t$ and discuss details of the three markets.

*The MP market:* is a random search market in which unemployed workers search for employment and new firms search for labor.\(^5\) Workers have one indivisible unit of labor service to sell to a firm during each period. Only unemployed workers search for employment.

\(^5\)See the work of Shi (2006) for a discussion of directed search and undirected search.
Firms are able to accommodate only one employee at any given time. If a worker and firm are combined, then they produce output $y$. The firm then sells the output in the LW market and the wage is paid in the AD market. Existing jobs are destroyed at an exogenous rate $\delta$ in the MP market.

The design of the MP market is as follows. First, the firms that are seeking to hire labor post a menu of wages that they are willing to pay to attract applicants. Workers subsequently choose to apply to one and only one firm while being ignorant of the decision of other workers. The uncoordinated nature of the job-search process implies that some firms attract multiple applicants, whereas others do not. The menu posted by a searching firm consists of two wage levels:

$$w = \begin{cases} w^h & \text{if only 1 worker approaches the firm} \\ w^l & \text{if more than 1 worker approaches the firm,} \end{cases}$$

where $w^h > w^l$. When more than one worker approaches a firm, only one of the applying workers enters an employment match. Each applicant has an equal probability of success. Competition between applicants for the sole available position results in a firm extracting all gains from a match. The low wage $w^l$ is the wage at which the successful applicant is indifferent between working and not working. By contrast, if only one worker approaches a searching firm, then we assume that this worker holds all of the wage determination bargaining power. Thus, the high wage $w^h$ in equilibrium is the wage at which the firm is indifferent between employing the worker and not employing the worker (and thus being inactive).

The LW market: firms and workers randomly search for one another to trade consumption goods. Firms that are matched in the MP market and have output $y$ to sell search for a worker to buy $y$ or at least a proportion of $y$. Workers, including those who are both matched and unmatched in the MP market, carry fiat money search for a firm to purchase consumption
goods to ensure that they can enjoy utility. Once paired in the LW market, the firm and the worker bargain over the terms of trade. The standard assumptions of monetary search theory are pivotal to the functioning of this market. Workers and firms are anonymous and meetings are quid pro quo.

The AD market: agents trade in the centralized (Walrasian) market. Consumption goods that are not sold in the LW market are transformed and transferred to the AD market by firms that either searched without success in the LW market or found a match in the LW market but did not sell their entire inventory. All incomes due by firms to workers (wages and dividends) are also paid in this market. Access to the centralized market during each period ensures the tractability of the model by causing the distribution of money holdings of workers to degenerate.\footnote{See Lagos and Wright (2005).}

Workers

A representative worker $h$ with employment status $e$ enters the AD market with real balance $z$. Within the AD market, $h$ maximizes utility with respect to the choice of consumption goods purchased, denoted by $x$, and the real balance to be carried into the next time period, which is denoted by $\hat{z}$, to solve

$$W^h_e(z) = \max_{x, \hat{z}} \{ x + \beta U^h_e(\hat{z}) \}$$

s.t. $x + \hat{z} = z + w_e + \Delta + \tau - T$

where $w_e$ is the real wage and

$$w_e = \begin{cases} 
  w^h & \text{if } e = 2 \\
  w^l & \text{if } e = 1 \\
  b & \text{otherwise.}
\end{cases}$$
Note that a wage is paid in the AD market even though matching occurs in MP. \( b \) is the unemployment benefit provided by the government in real terms, and \( b < w^i < w^h \). \( \Delta \) is the dividend income that is derived from firm profits, and \( T \) is a lump sum tax charged by the government. \( \tau \) is a lump sum payment injected by the central bank in each period.

Inserting the budget constraint into (1) yields

\[
W^h_e(z) = I_e + z + \max_{\hat{z}} \left\{ -\hat{z} + \beta U^h_e(\hat{z}) \right\}
\]  

(2)

where \( I_e = w_e + \Delta + \tau - T \) is the agent’s after-tax income conditional on \( e \). The assumption of quasi-linear utility is crucial here. From (2), it is clear that the optimal choice of \( \hat{z} \) is independent of \( z \) and \( I_e \) but does appear to depend on \( e \) through \( U^h_e \). However, as will be shown later, the LW utility function is independent of \( e \), which makes the derivative of \( U^h_e \) and, hence, \( \hat{z} \) independent of \( e \). Thus, every worker brings the same \( \hat{z} \) out of the AD market; thus, the model is analytically tractable.

In the LW market, agents interact in a decentralized market with anonymous bilateral matching. Once matched, workers and firms bargain over the terms of trade \((q,d)\), where \( q \) and \( d \) are the units of goods and real dollars exchanged, respectively. For \( h \) with money holdings \( z \) and employment status \( e \), the value function is

\[
V^h_e(z) = \alpha^h [v(q) + W^h_e(\rho z - \rho d)] + (1 - \alpha^h) W^h_e(\rho z),
\]

(3)

where \( \alpha^h \) is the probability of \( h \) meeting a firm. We multiply any real balances taken out of the LW market by \( \rho \) to obtain their value in the AD market. \( v(q) \) is the utility and we assume that \( v(0) = 0, v' > 0 \) and \( v'' < 0 \).

The probability of trade is a matching function determined by the measure of workers and firms, denoted by \( B \) and \( S \), respectively, who are active in the LW market. \( \alpha^h = M(B,S)/B \). Matching function \( M \) satisfies the usual matching friction assumptions including constant returns, \( \alpha^h = M(Q,1)/Q \), where \( Q = B/S \) is the LW market tightness. We assume that
$M(Q,1)$ is strictly increasing in $Q$, with $M(0,1) = 0$ and $M(\infty,1) = 1$, and $M(Q,1)/Q$ is strictly decreasing with $M(0,1)/0 = 1$ and $M(\infty,1)/\infty = 0$. As all workers participate in the LW market, $B = 1$. Firms that are matched in the MP market, those with $e = 1,2$, participate in the LW market; thus $S = 1 - u$, where $u$ is the unemployment rate. The matching function may be written as $\alpha^h = M(1,1 - u)$.

The analysis now progresses to the MP market, in which the key features of this paper are located. In the MP market, the value function for $h$ is constructed according to employment status

$$U_e^h(z) = \delta V_0^h(z) + (1 - \delta) V_e^h(z), \text{ where } e = 1, 2, \text{ and}$$

$$U_0^h(z) = \lambda_1^h V_1^h(z) + \lambda_2^h V_2^h(z) + (1 - \lambda_1^h - \lambda_2^h) V_0^h(z), \quad (4)$$

where $\delta$ is the exogenous rate at which MP matches are destroyed. For simplicity, we assume that if a match is destroyed, $h$ cannot gain employment until the next MP market meeting. $\lambda_1^h$ and $\lambda_2^h$ are the endogenous rates at which a match is a low-wage match and a high-wage match, respectively. The matching rates, $\lambda_1^h$ and $\lambda_2^h$, depend on $u$ and $v$, where $u$ is unemployment and $v$ is the number of vacancies posted by firms.

This description completes the outline of the representative worker’s one-period problem. The three value functions for $h$ can be collapsed into one Bellman equation. Substituting $V_e^h(z)$ from (3) into (4) and using the linearity of $W_e^h$, we have

$$W_e^h(z) = I_e + z + \beta E_\hat{e} \hat{W}_e^h(0) + \max \left\{ -\hat{z}(1 - \beta \hat{\rho}) + \beta \hat{\alpha}^h [v(\hat{q}) - \hat{\rho} d] \right\}, \quad (5)$$

where expectation $E$ is the expectation of the next period’s employment status $\hat{e}$. Note that the choice of $\hat{z}$ of worker does not depend on $e$.

**Firms**

We now consider the problem of the representative firm $f$. Before proceeding to the markets, we must emphasize that only firms with $e = 1, 2$ have any need for money. $f$ with
\( e = 1, 2 \) requires money to pay wages and dividends in the AD market. The money that is necessary for wage and dividend payments is raised each period in the LW and/or AD markets. Thus, \( f \) with \( e = 0, 1, 2 \) appears to have no need to carry money from the AD market. Considering the MP market first, the value function of \( f \) is

\[
U^f_e = \delta V^f_0 + (1 - \delta)V^f_e, \quad \text{where } e = 1, 2, \text{ and}
\]

\[
U^f_0 = \lambda_1^f V^f_1 + \lambda_2^f V^f_2 + (1 - \lambda_1^f - \lambda_2^f)V^f_0,
\]

where \( \lambda_1^f \) and \( \lambda_2^f \) are the endogenous rates at which active yet unmatched firms enter low- and high-wage contracts, respectively. Again, the matching rates \( \lambda_1^f \) and \( \lambda_2^f \) are functions of the MP market variables \( u \) and \( v \).

For \( f \) to participate in the LW and AD markets requires \( e = 1, 2 \). If \( e = 1, 2 \), then \( f \) carries output \( y \) to the LW market and sells a quantity \( q \), where \( q \in [0, y] \). The residual \( y - q \) is transformed into \( x = \zeta(y - q) \) units of the AD good and sold in the subsequent AD market with \( \zeta' \geq 0 \) and \( \zeta'' < 0 \). The opportunity cost of sale is defined as \( c(q) = y - \zeta(y - q) \).

Unless otherwise stated, we assume \( \zeta \) to be linear such that \( x = y - q \) and \( c(q) = q \). For a firm entering the LW market with \( e = 1, 2 \),

\[
V^f_e = \alpha^f W^f_e(y - q, \rho d) + (1 - \alpha^f)W^f_e(y, 0),
\]

where \( \alpha^f = M(B, S)/S = M(1/(1-u), 1) \) is the probability of \( f \) meeting and trading with a worker in the LW market. \( W^f_e(x, z) \) for \( e = 1, 2 \) is the value of entering the AD market with \( x \) units of the AD market consumption goods and \( z \) in cash receipts. The value function for \( f \) with \( e = 1, 2 \) entering the AD market is

\[
W^f_e(x, z) = x + z - w_e + \beta U^f_e.
\]
where \( w_e \) for \( e = 1, 2 \) is the real wage paid by \( f \). Simplifying, we obtain

\[
V_e^f = R - w_e + \beta [\delta \hat{V}_0^f + (1 - \delta) \hat{V}_e^f], \quad \text{for } e = 1, 2, \tag{6}
\]

where \( R = y + \alpha^f (\rho d - q) \) is the expected real revenue that an employed firm earns during each period. This expression clearly indicates that the expected real revenue is identical for \( e = 1, 2 \).

A firm with \( e = 0 \) has no wage obligations and has no output to sell. Such a firm is considered to be inactive. An inactive firm is able to enter the next MP market only if it pays a real cost, denoted by \( k \), incurred in the prior AD market. Thus,

\[
W_0^f = \max \left\{ 0, -k + \beta [\lambda_2^f \hat{V}_2^f + \lambda_1^f \hat{V}_1^f + (1 - \lambda_1^f - \lambda_2^f) \hat{V}_0^f] \right\},
\]

where \( \hat{V}_0^f = \hat{W}_0^f = 0 \) by free entry. In steady state, \( k = \beta (\lambda_2^f V_2^f + \lambda_1^f V_1^f) \). By (6), \( k \) can be expressed as

\[
k = \frac{\beta}{1 - \beta (1 - \delta)} [\lambda_1^f (R - w^f) + \lambda_2^f (R - w^h)]. \tag{7}
\]

**Government**

The role of government in this model is of minor importance. The government records a balanced budget in each period. The government collects lump sum taxes from workers, denoted by \( T \); pays an unemployment benefit to \( h \) with \( e = 0 \), denoted by \( b \); and prints money at rate \( \pi \) such that \( \dot{M} = (1 + \pi) M \), where \( \pi \) is the steady-state inflation. In the budget constraint for \( h \) in the AD market, \( \tau = \pi M/p \). The government pays its injection to workers directly. The government budget constraint is \( b u = T + \pi M/p \).

Because we focus on steady-state analysis, by the Fisher equation \( 1 + i = (1 + \pi)/\beta \), we can equivalently describe monetary policy in terms of setting the nominal interest rate \( i \) or the growth rate of money \( \pi \).
3.1 Equilibrium

3.1.1 The Goods Market

As noted previously, f and h meet and bargain bilaterally over \((q, d)\) in the LW market. Let 
\(\theta \in (0, 1)\) denote the bargaining power of \(h\). Thus, the Nash bargaining problem is

\[
\max_{q,d} [v(q) - \rho d]^{\theta}[\rho d - q]^{1-\theta}
\]

\[s.t. \ d \leq z, \ and\]

\[q \leq y.
\]

It is not difficult to show that \(d = z\) because it is costly to hold cash when we are not at
the Friedman rule. The first-order condition wrt \(q\) is

\[
\rho z = g(q, \theta), \text{ where } g(q, \theta) \equiv \frac{\theta v'(q)q + (1 - \theta)v(q)}{\theta v'(q) + 1 - \theta}.
\] (8)

We are now able to solve the equilibrium in the LW market. Using equation (5), which
is essentially the key equation of the LW market and then (i) inserting \(\hat{d} = \hat{z}\) and \(\partial \hat{q}/\partial \hat{z} = \hat{\rho}/g_1(\hat{q}, \hat{\theta})\) by virtue of (8), (ii) using the Fisher equation for the nominal interest rate to
eliminate \(1/(\beta \hat{\rho}) = 1 + i\), (iii) inserting the arrival rate \(\hat{\alpha}^h = M(1, 1 - \hat{u})\), and (iv) imposing
a steady state, we arrive at

\[
\frac{i}{M(1, 1 - u)} = \frac{v'(q)}{g_1(q, \theta)} - 1.
\] (9)

We refer to equation (9) as the LW curve. Simple conditions guarantee the existence of
a unique monetary equilibrium with \(q > 0\) given any \(u\). Routine calculations show that the
LW curve is downward sloping, for exactly the same reasons as cited in BMW: for higher
values of \(u\), there are fewer firms with output to sell in the decentralized goods market;
therefore, \(\alpha^h\) decreases. The terms of trade may also be adversely affected for workers as the

\[\text{See the work of Wright (2010) for a detailed analysis.}\]
number of firms selling consumption goods decreases. These effects are aggregated to reduce the demand for real money balances \( z \) and thus to reduce the amount of goods traded \( q \) via the bargaining solution.

Another important result from (9) is that \( q < q^* \) for all \( i > 0 \), where \( q^* \) is the efficient amount of goods traded, defined as \( v'(q^*) = 1 \). In other words, the steady state is efficient if and only if \( i = 0 \) and \( \theta = 1 \). Intuitively, there are two types of inefficiencies. First, workers do not always find a trading partner in the goods market. When it is costly to hold money, workers choose to carry less than the efficient amount. In the second type of inefficiency, the choice of firms to bargain away part of the trading surplus also reduces the incentive to hold money. Invoking Proposition 1 of BMW, for \( i > 0 \), the LW curve slopes downwards in \((u, q)\) space.

### 3.1.2 The Labor Market

To solve the labor market equilibrium condition (7) in terms of \((u, q)\), we must first calculate the two wage levels \( w^h \) and \( w^l \). As previously stated, the high wage in equilibrium is the wage at which the firm is indifferent between employing a worker and not employing a worker: \( V^f_2 = V^f_0 \). By contrast, the low wage is the wage at which a successful applicant is indifferent between employment status \( e = 1 \) and \( e = 0 \): \( V^h_1 = V^h_0 \). These two conditions allow us to solve \( w^h \) and \( w^l \) as a function of \((u, q)\). The results are

\[
w^h = R = y + \alpha^f [g(q, \theta) - q], \quad \text{and} \quad (10)
\]

\[
w^l = \frac{1 - \beta (1 - \delta)}{1 - \beta (1 - \delta - \lambda^h_2)} b + \frac{\beta \lambda^h_2}{1 - \beta (1 - \delta - \lambda^h_2)} w^h. \quad (11)
\]

Substituting the two wage levels expressed in (10) and (11) into the free entry condition (7) and then inserting \( \rho d - q = g(q) - q \) by virtue of (8), we obtain

\[
\frac{k}{\beta} = \frac{\lambda^f_1}{1 - \beta (1 - \delta - \lambda^h_2)} [y + \alpha^f (g(q, \theta) - q) - b]. \quad (12)
\]
This equation is the general expression for the MP curve. To map the curve in \((u, q)\) space, we must define the matching functions \(\lambda^f_1, \lambda^f_2, \lambda^h_1\) and \(\lambda^h_2\).

Consider a worker who is unemployed. If \(v\) vacancies are posted in the labor market, then the probability of the worker approaching a certain searching firm is \(1/v\). The probability that the worker does not approach a certain firm is \(1 - 1/v\). Then, the probability that the firm is not approached by any unemployed workers is \((1 - 1/v)^u\). The probability that exactly one worker approaches the firm in the labor market, defined as the matching rate \(\lambda^f_2\), is \(\phi(1 - 1/v)^{u-1}\), where \(\phi = u/v\) is defined as labor market tightness. \(\lambda^f_1\), the probability that more than one worker approaches the firm in the labor market, is \(1 - (1 - 1/v)^u - \phi(1 - 1/v)^{u-1}\).

For an unemployed worker to enter a high-wage contract, the worker must be the sole applicant at the firm to which he or she applies to. The probability that the worker is the sole applicant is the probability that no other workers approach the firm. Hence, \(\lambda^h_2\) is equal to \((1 - 1/v)^{u-1}\). The probability of the worker entering a low-wage match, \(\lambda^h_1\), is the probability of at least one other worker applying at the same firm, \(1 - (1 - 1/v)^{u-1}\), divided by the number of applicants. The average number of applicants at a firm is best approximated by \(\phi\). Therefore, \(\lambda^h_1 = (1 - (1 - 1/v)^{u-1})/\phi\).

This paper considers the effect of inflation on wage inequality in a large economy by holding \(\phi\) constant and examining the case in which \(v\) is a large but finite number. In this type of environment, the economy can be closely approximated by the limit economy in which \(v \to \infty\).\(^8\) Using the rule that \(\lim_{v \to \infty} (1 + x/v)^v = e^x\), the labor market matching functions are,

\[
\begin{align*}
\lambda^f_2 &= \lim_{v \to \infty} \phi(1 - 1/v)^{v\phi-1} = \phi e^{-\phi}, \\
\lambda^f_1 &= \lim_{v \to \infty} 1 - \phi(1 - 1/v)^{v\phi-1} - (1 - 1/v)^{v\phi} = 1 - \phi e^{-\phi} - e^{-\phi}, \\
\lambda^h_2 &= \lim_{v \to \infty} (1 - 1/v)^{v\phi-1} = e^{-\phi}, \text{ and} \\
\lambda^h_1 &= \lim_{v \to \infty} \frac{1 - (1 - 1/v)^{v\phi-1}}{\phi} = \frac{1 - e^{-\phi}}{\phi}. \\
\end{align*}
\]

\(^8\)For a more detailed analysis, see Julien et al (2000).
Finally, the Beveridge curve \((1 - u)d = u(\lambda_1^b + \lambda_2^b)\) allows us to solve for and insert \(v = v(u)\) into the above matching functions. It is useful to note here that the Beveridge curve is downward sloping; that is, \(\partial v/\partial u < 0\). This condition yields \(\partial \phi/\partial u > 0\). A higher rate of unemployment, which simultaneously reduces the number of job vacancies available in the steady state, increases labor market tightness.

Substituting the matching functions (13) into equation (12), we obtain the final specification of the MP curve:

\[
\frac{k}{\beta} = \frac{1 - e^{-\phi} - \phi e^{-\phi}}{1 - \beta(1 - \delta - e^{-\phi})} \left\{ y + M\left(1 - u, 1\right)[g(q, \theta) - q] - b \right\},
\]

where \(\phi\) is now \(u/v(u)\). This MP curve determines \(u\) by taking the value of money \(q\) as given. Here, we assume that \(k < \beta[y - b + g(q^*) - c(q^*)]/[1 - \beta(1 - \delta)]\) because otherwise the labor market would simply shut down.

Differentiating (14) provides the solution that \(dq/du\) is negative. The MP curve slopes downward in \((u, q)\) space. The intuition is simple: there are three general effects of an increase in \(u\). First, it is easier for firms to hire, \(\lambda_1^f\) increases; second, it is more difficult for workers to become hired, \(\lambda_2^h\) decreases; third, it is easier for firms to compete in the LW market, \(\alpha^f\) increases. These three effects encourage the entry of firms. Thus, the equilibrium value of \(q\) must be reduced to ensure that the free entry condition holds.

### 3.2 General Equilibrium

Figure 3 maps the LW and MP curves in a box \(B = [0, 1] \times [0, q^*]\) in \((u, q)\) space. The LW curve enters \(B\) from the left at \((0, q_0)\), where \(q_0 \leq q^*\), and it exits at \((1, 0)\). Further calculations confirm that LW is concave in \(B\). The MP curve enters \(B\) from the top at \((u_0, q^*)\). If \(k < \beta(y - b)/(1 - \beta(1 - \delta))\), then it exits from the bottom at \((u_1, 0)\) (see the curve labelled MP1). In this case, a non-monetary equilibrium exists at \((u_1, 0)\), at least one monetary equilibrium also exists. If \(k \geq \beta(y - b)/(1 - \beta(1 - \delta))\), then the MP curve
exits B from the right at \((1, q_1)\). In this case, there exists a non-monetary equilibrium at \((1,0)\) and, depending on parameter values, there may also exist monetary equilibria (see the curves labelled MP2 and MP3). Generally, we do not have uniqueness. Monetary and non-monetary equilibria may coexist. However, it is possible for monetary equilibrium to be unique, as in the calibrations below. When we obtain the equilibrium values for \(u\) and \(q\), all other endogenous variables, including the real wages \(w^f\) and \(w^h\), can be easily recovered.

Before considering the effect of inflation on wage inequality, we must analyze the effects on the steady-state equilibrium values of \(u\) and \(q\) as inflation increases. Conveniently, any change in inflation will not affect the MP curve. The LW curve shifts toward the origin as the rate of inflation increases. Therefore, an increase in the rate of inflation causes a decrease in \(q\) and an increase in \(u\).

**Proposition 1** Steady-state monetary equilibrium exists. If \(k < \beta(y - b)/(1 - \beta(1 - \delta))\), then there exists at least one monetary steady state in which \(q < q^*\) and \(0 < u < 1\). If \(k \geq \beta(y - b)/(1 - \beta(1 - \delta))\), then there is a non-monetary steady state at \((1,0)\), and depending on parameter values, monetary steady states may also exist. If the monetary steady state is unique, then an increase in \(i\) causes a decrease in \(q\) and an increase in \(u\).
3.3 Results

We now consider the effect of inflation on wage inequality. Formally, we define wage inequality as the ratio between the high wage \( w^h \) and the low wage \( w^l \).

\[
WI = \frac{w^h}{w^l}.
\]

(15)

Differentiating (15) wrt \( i \) yields mixed results. However, we can show that with reasonable parameter values,

\[
\frac{\partial (WI)}{\partial i} > 0.
\]

**Proposition 2** An increase in the rate of inflation increases real wage inequality. Moreover, although both high and low wages decline with inflation, the negative effect is stronger on the low wage than on the high wage.

The intuition for this result is simple. From (10), it is evident that when inflation increases, the quantity of goods traded in the LW market declines (Proposition 1), which leads to a decrease in trade surplus \( g(q) - q \). As this surplus decreases, firms post fewer vacancies in the MP market, and the steady-state rate of unemployment increases. The labor market becomes tighter: the \( u/v \) ratio increases. We term this effect a spillover effect (a spillover from the goods market to the labor market). This effect in turn increases the ease with which firms (sellers) can meet workers (buyers) in the LW market; thus, \( \alpha^f \) increases. However, as suggested by the results, with reasonable parameter values the decrease in \( g(q) - q \) dominates the increase in \( \alpha^f \); therefore, the overall effect on the real profits of firms from LW trades is negative, which explains why the high wage decreases in reaction to increases in inflation.

With regard to low wages, from (11), it is evident that the low wage is a weighted average of workers’ outside value \( b \) and the high wage \( w^h \) (which is equal to a firm’s revenue \( R \)). As discussed above, when inflation increases, the spillover effect leads to a tighter labor market.
This effect increases the average number of applicants for each vacant firm. Because of the lower likelihood of locating a high-paying job, $\lambda^h_2$ decreases. The decline in $\lambda^h_2$ affects the weights of $b$ and $w^h$ differently. This decline increases the weight of $b$ and thus moves the low wage toward the value of $b$ in the episodes of high inflations. However, the decrease in $\lambda^h_2$ lowers the weight that is assigned to $w^h$, which, in addition to a reduced $w^h$, places further downward pressure on the low wage. As the results suggest, with reasonable parameter values, the increase in the weight of $b$ is insufficient to compensate for the decrease in $w^h$ and the decrease in the weight of $w^h$; therefore, low wages always decline more than high wages. Thus, an increase in inflation leads to an increased wage inequality.

4 Numerical Analysis

This section calibrates the model constructed in Section 3 to data for the United States. Our objective is to analyze the extent to which our model can explain the observed effect of inflation on low wage $w^l$, high wages $w^h$, and the resulting wage inequality as measured by $w^h/w^l$.

Following BMW, the numerical analysis of this section uses the following specifications. In the LW market, the utility function assumes the functional form of $v(q) = Aq^{1-a}/(1-a)$, and the transformation cost function is $c(q) = q^\gamma$. The matching function is assumed to be $M(B, S) = BS/(B + S)$. Given these specifications, the parameters to be determined include preference parameters ($\beta$, $A$, $a$), the technology parameters ($y$, $k$, $\delta$, $\gamma$, $\theta$) and the policy parameter $b$.

4.1 Parameterization

The model period is set to be one quarter. The calibration targets that we choose are standard in the literature. As shown in Table 3, we aim to reproduce the main features in the labor market, goods market and money market in the long term. The discount factor $\beta$
is set to match the annual real interest rate, which is 4.8%. In the model, the real demand for money $M/(pY)$ is measured by

$$\frac{M}{pY} = \frac{g(q)}{(1-u)\{\alpha f[g(q) - c(q)] + y\}}.$$ 

The scale parameter $A$ and curvature parameter $a$ in the utility function jointly determine the value of $M/(pY)$ through the function of $g(q)$; therefore, these two parameters are set to target the annual average real demand for money and the reaction to changes in the nominal interest rate (elasticity), which are 0.179 and $-0.556$, respectively, as reported by BMW. Because we focus on the steady-state analysis, the productivity in a formed match is normalized to one. The flow cost of posting a vacancy $k$ is determined to match the average unemployment, which is 5.67 percent for the period from 1951 to 2005. Because there is no endogenous job dissolution in the model, the exogenous job separation rate $\delta$ is set to 0.04. In other works, Shimer (2005) selects the value of 0.033, whereas BMW (2011) sets this value to 0.05. Here, we use the average of these two values of $\delta$. We find that the effects of inflation that are generated by the model become larger when the value of $\delta$ decreases. The curvature parameter in the cost function $\gamma$ is normalized to one in the baseline simulation, and will change to other values in the robustness checks. The bargaining power of buyers in the LW market $\theta$ becomes apparent in the function of $g(q)$, which determines the profit rate in the LW market. Hence, the value of $\theta$ is chosen to match the mark-up ratio, which is 30 percent, as summarized by Faig and Jerez (2005). In the model, the mark-up ratio can be measured by

$$\text{markup} = 100 \left( \frac{g(q)}{c'(q) q} - 1 \right).$$

For policy parameters, the UI benefit $b$ is set to be 0.4, which is the value of the replacement ratio that is widely used in the literature (for example, in Shimer (2005)).

Table 3 reports the calibration targets. The values $\{\beta, y, \delta, \gamma, b\}$ follow directly from the stated targets in the table. The remaining parameters $\{A, a, k, \theta\}$ are obtained from the
following iterative procedures. The steady-state values of \( \{u, q\} \) are the solution to the system of equations that contain (9) and (14). With some initial estimates for \( \{A, a, k, \theta\} \), we fit the observed nominal interest rate over the sample period into the system of equations, solve for the equilibrium values of \( \{u, q\} \), and verify that the predictions of the model match the targets that are stated in Table 3. If the predictions do not match, then the initial estimate is revised, and the above process is repeated until the predictions from the calibrated model appropriately match all of the targets.

### Table 3

**Baseline Parameterization**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Target Descriptions</th>
<th>Target Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>( \beta ) annual real interest rate</td>
<td>0.048</td>
</tr>
<tr>
<td>Scale parameter in ( v(q) )</td>
<td>( A ) real demand for money (annual) in BMW</td>
<td>0.179</td>
</tr>
<tr>
<td>Curvature parameter in ( v(q) )</td>
<td>( a ) elasticity of money demand wrt ( i ) in BMW</td>
<td>-0.556</td>
</tr>
<tr>
<td>Productivity in a formed match</td>
<td>( y ) normalization</td>
<td>1</td>
</tr>
<tr>
<td>Flow cost of posting a vacancy</td>
<td>( k ) average unemployment rate (quarterly)</td>
<td>5.67%</td>
</tr>
<tr>
<td>Separation rate</td>
<td>( \delta ) job separation rate in BMW</td>
<td>0.04</td>
</tr>
<tr>
<td>Curvature parameter in ( c(q) )</td>
<td>( \gamma ) normalization</td>
<td>1</td>
</tr>
<tr>
<td>Bargaining power in LW market</td>
<td>( \theta ) Mark-up ratio in Faig and Jerez (2005)</td>
<td>0.30</td>
</tr>
<tr>
<td>UI benefits</td>
<td>( b ) replacement ratio</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 4 reports the calibrated values of parameters \( \{A, a, k, \theta\} \). The values in the first column correspond to the baseline calibration as described above. The remaining four columns correspond to the alternative calibrations in which the targets for the key parameters \( b, \gamma \) and \( \theta \) are changed. The values of \( b, \gamma \) and \( \theta \) are crucial for understanding the effects of inflation on wage inequality. Equation (11) suggests that with a smaller \( b \), the same change in inflation imposes a greater downward pressure on low wage and thus results in increased
wage inequality. When we increase the value of $\gamma$, it becomes more costly to produce goods in the goods market; with all else equal, this costliness reduces the real profits of firms and generates a stronger spillover effect into the labor market. Finally, a increase in the mark-up ratio in the goods market reduces the quantity of goods traded in the LW market $q$, which reduces the incentives of firms to post vacancies and translates into a higher unemployment rate. Because of the importance of these three parameters, we depart from the baseline calibration by changing their targets in the robustness check.

In the second column of Table 4, which is termed Benefits, $b$ is set to be zero. In the third column, which is called Curvature, $\gamma$ is set to 1.1. In the fourth column, which is termed Markup, $\theta$ is determined to match the mark-up rate of 0.4 rather than 0.3. In the last column, which is labeled All, we simultaneously change the targets for all three parameters. Although these alternative targets are arbitrarily set, they are useful to illustrate how the results depend on the parameter values.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
</tr>
<tr>
<td>$a$</td>
<td>0.3048</td>
</tr>
<tr>
<td>$A$</td>
<td>1.1142</td>
</tr>
<tr>
<td>$k$</td>
<td>1.4633</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.6042</td>
</tr>
</tbody>
</table>

4.2 Results

Using the calibrated parameters, the first row in Table 5 reports the wage inequality predicted by the model. In the baseline model, the value is 1.16, which is approximately 75 percent of the empirical counterpart as calculated in Section 2 ($1.16/1.54 = 0.75$). The predicted value is fairly close to the observed wage inequality when all alternative targets are considered ($1.28/1.54 = 0.83$). This result proves that the calibrated model is suitable for studying the
effects of inflation on wage inequality.

The remaining three rows in Table 5 summarize the effects of a doubled inflation rate on low wages $w^l$, high wages $w^h$ and wage inequality $w^h/w^l$ in various cases as considered in the calibrations. To facilitate the comparison, we report their empirical counterparts ($WI_{75} = wage_{75}/wage_{25}$) in the last column of the table.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Benefits</th>
<th>Curvature</th>
<th>Markup</th>
<th>All</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w^h/w^l$</td>
<td>1.1620</td>
<td>1.2831</td>
<td>1.2868</td>
<td>1.1637</td>
<td>1.2803</td>
<td>1.54</td>
</tr>
<tr>
<td>$\Delta w^l$ (%)</td>
<td>-3.08</td>
<td>-3.39</td>
<td>-4.99</td>
<td>-3.97</td>
<td>-5.66</td>
<td>-3.83</td>
</tr>
<tr>
<td>$\Delta w^h$ (%)</td>
<td>-2.94</td>
<td>-2.93</td>
<td>-4.30</td>
<td>-3.78</td>
<td>-4.90</td>
<td>-2.61</td>
</tr>
<tr>
<td>$\Delta (w^h/w^l)$ (%)</td>
<td>0.14</td>
<td>0.46</td>
<td>0.69</td>
<td>0.19</td>
<td>0.76</td>
<td>1.23</td>
</tr>
</tbody>
</table>

Table 5 clearly shows that the model-predicted effect on the levels of wages and wage inequality are qualitatively consistent with the empirical evidence. A 100 percent increase in inflation reduces both low and high wages but has a stronger effect on low wages, which leads to increased wage inequality. For instance, in the baseline case, the low wage decreases by 3.08%, whereas the high wage decreases by only 2.94%. This result indicates that wage inequality increases by 0.14% in response to the change in inflation. This finding remains robust in all cases.

Quantitatively, in the baseline calibration, the predicted responses of low wages account for 80% of the observed movement (−3.08% vs. −3.83%), and the predicted reactions of high wages is also fairly close to its empirical counterpart (−2.94% vs. −2.61%). Departures from the baseline case by lowering the UI benefits and increasing the convexity of the cost function and the mark-up ratio contribute to enhancing the model fit. The table shows that when all modifications are made, the predicted responses of wage inequality account
for approximately two-thirds of the observed adjustments in wage inequality as measured by $WIL_75$ ($0.76/1.23 = 0.62$).

5 Conclusion

In this paper, we explore the relationship between inflation and residual real wage inequality. Using the CPS data for the 1994 - 2008 period, we find that inflation has a positive effect on wage inequality among homogenous workers, and this influence is primarily observed in terms of a negative effect on low wages. To explain these facts, we develop a model based on the search-theoretic monetary framework in BMW that features a variation of the wage-posting model in the work of Julien et al (2000). Our model incorporates a capacity constraint and an uncoordinated job search, which are realistic features of economies.

We show that inflation increases wage inequality, and the intuition is simple: inflation reduces the real profits of firms, which is equal to the high wage. Because low wage in equilibrium is a weighted average of the high wage and the UI benefits, lower profits also reduce the low wage. Furthermore, inflation causes firms to post fewer vacancies, which translates into a tighter labor market. As a result, it is less likely that unemployed workers will find high-wage jobs. This outcome reduces the weight of the high wage and increases the weight of the UI benefits, which places further downward pressure on the low wage. When the model is tested with data for the United States, the qualitative predictions of the model are consistent with the empirical evidence. Quantitatively, our model can explain approximately two-thirds of the observed adjustments in wage inequality.

Acknowledgements

Min Zhang appreciates the financial support from the National Natural Science Foundation of China (Grant No. 71203132) and the Leading Academic Discipline Program, 211 Project for Shanghai University of Finance and Economics (the 4th phase).
References


Appendix


The Current Population Survey (CPS) is a monthly survey of approximately 50,000 workers conducted by the Bureau of the Census for the Bureau of Labor Statistics. The survey has been conducted for more than 50 years. To be eligible to participate in the CPS, individuals must be at least 15 years of age and must not serve in the Armed Forces. In general, one person ("reference person") responds on behalf of all eligible members of the family.

The original data set contains more than 200,000 person-level observations per month. To create our sample, we exclude all individuals who report zero hourly wages in their main job and individuals whose hourly wages are top-coded (i.e., greater than $99). We then obtain with approximately 2,000 individual observations per month during the period from 1994 to 2008.

B: Proofs

Proof of Proposition 1. A differentiation of (9) wrt the nominal interest rate, \(i\), shows that \(\frac{dq}{di} < 0\) for a given \(u\). Both the MP and LW curves are downward sloping. Given this result and considering that any change in \(i\) will not shift the MP curve (see equation (14)), we infer that an increase in \(i\) (which is equivalent to an increase in the inflation rate) will cause a decrease in \(q\) and an increase in \(u\).

Proof of Proposition 2. Because differentiating (15) does not provide us closed-form solutions, we divide the analysis into two steps. First, we examine an alternative expression of (15) to gain a better understanding of how inflation affects wage inequality and wage levels. We define \(\widetilde{W}\) as

\[
\widetilde{W} = \frac{w^h - b}{w^l - b}
\]

Substituting away \(w^h\) and \(w^l\) using equations (10) and (11) and then simplifying the results, we obtain

\[
\widetilde{W} = \frac{1 - \beta(1 - \delta - e^{-\phi})}{\beta e^{-\phi}}.
\]

It is immediately obvious that \(u\) is the only steady-state variable to appear in (16). To consider the effects of inflation on \(\widetilde{W}\), we only need to consider the effect of increased inflation on the steady-state equilibrium value of \(u\). It is routine to show that

\[
\frac{\partial \widetilde{W}}{\partial i} > 0.
\]

In the second step, we use the above result to solve for \(\partial(WI)/\partial i\). There are two possible cases with regard to the effects on wage levels and wage inequality.

Case 1. \(w^h\) decreases with inflation. Because \(\partial(WI)/\partial i > 0\), it is suggested that \(w^l\) must also decrease. The effect on \(WI\) (which is defined as \(w^h/w^l\)) is ambiguous. To more closely
examine this relationship, we rewrite $\widetilde{WI}$ as follows

$$\widetilde{WI} = \frac{w^h - b}{w^l - b} = \frac{w^h}{w^l} - \frac{b}{w^l}. \quad (17)$$

Rearranging and differentiating (17) wrt. $i$ yields

$$\frac{\partial (\frac{w^h}{w^l})}{\partial i} = (1 - \frac{b}{w^l}) \frac{\partial (\widetilde{WI})}{\partial i} - (\widetilde{WI} - 1) \frac{\partial (\frac{b}{w^l})}{\partial i} \quad (18)$$

Because $b/w^l \leq 1$ and $\widetilde{WI} \geq 1$ and because $\partial (\widetilde{WI})/\partial i > 0$ and $\partial (b/w^l)/\partial i > 0$, we still obtain ambiguous results in terms of $\partial (w^h/w^l)/\partial i$. However, (18) clearly indicates that when the value of $b$ is much smaller than $w^l$ (which is true in the data and in the calibration), $1 - b/w^l$ becomes larger and $\partial (\widetilde{WI})/\partial i$ then becomes positive. The positive effect is maximized when $b = 0$. Hence, we conclude that if $\partial (w^h)/\partial i < 0$, then $\partial (w^l)/\partial i < 0$ and $\partial (w^h/w^l)/\partial i > 0$.

Case 2. $w^h$ increases with inflation. Now, $w^l$ can increase or decrease. If $w^l$ decreases, then $\partial (w^h/w^l)/\partial i > 0$. In the case in which $w^l$ increases, (18) suggests that $\partial (w^h/w^l)/\partial i > 0$ is still true. Therefore, we conclude that if $\partial (w^h)/\partial i < 0$, then $\partial (w^l)/\partial i < 0$ or $\partial (w^l)/\partial i > 0$ and $\partial (w^h/w^l)/\partial i > 0$. However, as explained earlier, with plausible parameter values, real profits of firms $R$ always decrease with inflation; therefore, $w^h$ also declines with inflation. Case 2 is irrelevant.

In summary, we show that if $\partial (w^h)/\partial i < 0$, then $\partial (w^l)/\partial i < 0$ and $\partial (w^h/w^l)/\partial i > 0$. 

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