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Authors: hansenj@rba.gov.au and grossi@rba.gov.au
Abstract

This paper studies the effect of a shock to resource prices in a small open economy and where the stock of natural resources is endogenous. In particular, we model a resources sector where firms have access to an exploration technology, facilitating the accumulation of new reserves, and account for the fact that resources are depletable.

Using this richer production structure, we show that the effects of a resource price shock on resource-specific investment, labour utilisation and extraction are all amplified in the presence of endogenous reserves. And that this amplification has spillover effects for broader economy activity including non-traded production, non-resource exports, and consumption. However, for key price measures, such as domestic inflation, the real exchange rate and domestic interest rates, we find that incorporating endogenous reserves does not fundamentally change the propagation of resource price shocks.

JEL Classification Numbers: Q33, F41
Keywords: natural resources, small open economy
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1. Introduction

Large movements in commodity prices over the past decade have spurred a renewed interest in the effects of commodity price shocks on a small open economy. One group of commodities that has received increasing attention, and especially in the Australian case, are non-renewable resource commodities such as iron-ore and coal. This paper studies how an alternative approach to modelling resource firms’ decision problems can affect the propagation of a commodity price, or more specifically a resource price, shock.

One method used to study the effects of a resource price shock is to integrate a natural resources sector within a dynamic stochastic general equilibrium (DSGE) model. By incorporating natural resources in such a model, researchers have been able to explore the implications of changes in resource prices for the domestic economy and domestic macroeconomic policy. This question is thought to be especially relevant in the context of small open economies, and particularly those with a large concentration of their exports in natural resources such as oil, coal, gas and other minerals.¹

A common assumption used in existing literature is to assume that the domestic economy’s stock of resource reserves is exogenous. That is, when choosing to extract resources, firms do not account for the fact that extracting resources today reduces the amount of resources available for future extraction (depletion). In addition, firms are unable to invest in a technology that changes the level of available reserves, for example through exploration and the discovery of new reserves or extensions to existing deposits. Examples of the “exogenous reserves” approach to modelling the natural resources sector include Bodenstein, Erceg and Guerrieri (2011), Bems and de Carvalho Filho (2011), Dib (2008), Garcia and Gonzalez (2010), Lama and Medina (2012), and Natal (2012). Similar

¹ Throughout the paper we use the term natural resources as being synonymous with non-renewable resources and abstract from renewable natural resources.
abstractions are also common in a number of DSGE models developed within a number of central banks including Australia (Jäskelä and Nimark 2008), Canada (Murchison and Rennison 2006), Spain (Andrés, Burriel and Estrada 2006) and New Zealand (Lees 2009).

Although a useful simplifying assumption for studying many questions, a limitation of this approach is that there is nothing inherently natural-resource like in the decision program for resource producers. An important question is to what extent does the assumption of exogenous reserves affect our understanding of the propagation of resource price shocks? Would these responses look especially different once one allows for endogenous reserves in the form of exploration and depletion? This paper seeks to answer these questions with specific reference to the effects of a resource price shock.

Our findings suggest that allowing for endogenous reserves does have substantial effects on the magnitude and persistence of the resources sector response to a price shock in both partial and general equilibrium. The mechanism at the core of our model, the ability to accumulate newly discovered reserves through exploration, implies that firms respond to a price shock by increasing both extraction and exploration. Exploration resulting in newly discovered reserves in turn leads to a permanent increase in firms’ future extraction possibilities set, and firms expand their labour utilisation, investment activity and extraction by more when the stock of natural resources is assumed to be endogenous.

We find that this greater expansion in the resources sector has implications for the domestic allocations of goods, with greater inter-sectoral reallocation of goods in response to a resource price shock. When comparing models with endogenous and exogenous reserves respectively, we find that the declines in non-traded good allocations, both through domestic consumption and as input in the production of exports, are larger when an endogenous stock of reserves is assumed. The fall in domestic production is, nevertheless, absorbed by the expansion of the resources sector and so the net effect on domestic output is small.

When comparing key price measures, including domestic inflation, the real exchange rate and domestic interest rates, we find that the effects of a resource price shock are relatively similar irrespective of whether an endogenous or exogenous stock of natural resources is assumed. This suggests that the
standard approach of assuming exogenous resources can still provide a useful approximation for quantifying price effects associated with a resource price shock.

We first discuss our empirical motivation, Section 2, and highlight that the stock of natural reserves in Australia has not been constant over time and does appear to be correlated with changes in resource prices. We then turn to the theoretical implications of an endogenous resource reserves model, examining the mechanism of interest in both partial (Section 3) and general equilibrium (Section 4). To parameterize the economies that we study, we use a simple empirical VAR that only imposes weak restrictions consistent with those implied by the models that we discuss in general equilibrium.

In terms of related literature, the only paper that the authors are aware of that nests an endogenous reserve model in a DSGE model is by Veroude (2012) who studies a closed economy. There is, however, a voluminous literature on the importance of exploration and depletion in the context of the standard natural resources literature that studies optimal extraction, investment and exploration decisions, but not within the context of a small-open economy. A non-exhaustive list of useful references includes Pindyck (1978), Reiss (1990), Heal (1993), Sweeney (1993) and Bohn and Deacon (2000). There is also an extensive literature studying the comparative statics of general equilibrium models with multiple sectors including resources (see for example Gregory (1976) and Corden (2012)), although these models do not incorporate expectations or dynamics.

2. Motivation

Figure 1 highlights some of the key developments in the Australian resources sector since 1976. The variables graphed in Figure 1 include a measure of average prices in the sector, average production (or extraction), real exploration expenditure, and the average stock of reserves. Reserves for each resource commodity are measured to include both economically demonstrated reserves – reserves considered to be economically profitable for extraction purposes – and sub-economic reserves – reserves not considered to be currently viable but that may become viable in the future with higher resource prices or a cost-reducing advance in technology. The resources included in these measures are iron-ore, coal,

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2 All averages - prices, production and reserves, are export-weighted geometric averages, where the export weights used are fixed at the sample averages for 1976 to 2011.
coal, oil (including crude oil, condensate and LPG), natural gas, five base metal ores (bauxite, copper, lead, nickel and zinc), and gold. Together these resources accounted for approximately 88 per cent of total resource exports, and 66 per cent of total goods exports in 2011/12.

Summarising the main stylised facts:

1. Resource prices trended down for much of the sample but boomed around the turn of the millennium reflecting strong commodity demand particularly from India and China;

2. Production growth has been most rapid in the mid to late 1980s but growth has since stabilised from around 1992

3. Real exploration peaked in the early 80’s and then declined for much of the period in which real resource prices fell. From around 2000, however, real exploration activity began to grow rapidly as the boom in resource prices became sustained;

4. The pace of growth in reserves has generally slowed over the 20 years between 1980 and 2000 but has then accelerated from the early to mid 2000s suggesting that at least some of the pick up in exploration has also resulted in the discovery of new reserves or extensions to existing deposits.

These facts suggest that reserves and exploration are correlated with resource prices. And so allowing for endogenous reserves could be informative for understanding how the resources sector responds to a price shock.

To provide evidence on the broader effects of a resource price shock on the rest of the domestic economy, we use a simple structural VAR, identified under the assumption that shocks to resources prices are contemporaneously uncorrelated with domestic variables. Using the theoretical model we discuss below as motivation for the specification of the VAR, the variables included are natural

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3 See Geoscience Australia (2012) for further discussion on the classification of reserves. From 1992 onwards, reserves are measured using the sum of economically demonstrable reserves and sub-economic reserves. Prior to 1992, all reserves measures are based on economically demonstrated reserves only and are spliced to the post 1992 series. Estimates for production and reserves in 2011 are inferred using the growth rates implied in ABS data.
Figure 1: Developments in the Australian Resources Sector

Sources: ABARES; ABS; Bloomberg; Geoscience Australia; Global Financial Data; IMF; USGS; Authors’ calculations

Notes: Resource prices, production and reserves are calculated using export-weighted geometric means. Exploration expenditure comprises all categories of mineral and petroleum expenditure.

resource prices, the real TWI, inflation, the ratio of non-mining GDP to the stock of natural reserves, and the ratio of resources sector capital expenditure to the stock of natural reserves. We choose to deflate non-mining GDP and resource specific capital expenditure by reserves because this is important for the ability of the VAR to be able to recover the effects of a resource price shock when estimated on data simulated from our theoretical model.\(^4\)
Figure 2 reports the impulse response functions (IRFs) associated with a one per cent resource price shock as estimated by our empirical VAR on annual data. We also report the ninety five percent (asymptotic) confidence intervals associated with the empirical VAR, and the model-IRFs that best match those of the empirical VAR when using a Generalised Method of Moments (GMM) estimator (discussed further in Section 4.7).

According to the empirical VAR, resource price shocks are very persistent and have significant effects on both the resource sector and the broader economy. In particular, the empirical VAR highlights that a resource price shock leads to a persistent real appreciation of the exchange rate, an increase in inflation, an increase in the resource capital expenditure relative to reserves. A small, though not statistically significant, decline in the ratio of non-mining GDP to reserves is also observed.

Interestingly, the model is able to reproduce these response functions quite well when considering the sign, amplitude and persistence. The main exception is for the real exchange rate. Although our model is able to reproduce an appreciation, it is not sufficiently large nor persistent when compared with the response identified in the empirical VAR. Nevertheless, in view of the models overall ability to match the empirical-VAR IRFs, we use the GMM estimates to help parameterise our models in both the partial and general equilibrium analyses that follow.

3. Modelling Natural Resources in Partial Equilibrium

3.1 The Resources Sector

Our model of the resources sector draws from the work of Bohn and Deacon (2000). These authors allow for both endogenous exploration and depletion, and use an approach that naturally lends itself to incorporating the

---

4 More specifically, we use a VAR in the five mentioned variables with a single lag and use a simple Cholesky decomposition ordering resource prices first. This is equivalent to estimating the VAR equation by equation and using the residuals of the resource price equation as an instrument for contemporaneous resource prices, and so only the resource price shock is consistently identified in the system. We also include a deterministic time trend and constant in the VAR, though similar qualitative estimates are obtained using either a VAR on HP-filtered data or when applying the VAR on first differenced data.
Figure 2: Empirical and Model Impulse Response Functions to a 1 per cent increase in resource prices

- **Natural Resource Prices**
  - Model
  - VAR

- **Resources Capital Expenditure to Reserves**

- **Real Exchange Rate**

- **Non-Mining GDP to Reserves**

- **Inflation**
resources sector within a small-open-economy DSGE model. For the structure of the resources sector, the main assumptions we use are:

1. All resources are exported at prices that are taken as given by resource firms (that is, the resources market is globally competitive);\(^5\)

2. Resource firms can choose to extract a commodity from existing reserves and can engage in costly exploration activity to discover new reserves;

3. Resource firms use domestic labour, imported capital, and reserves to extract their natural resource;

4. All resource firms are identical in terms of their access to exploration and extraction technologies.

These assumptions are designed to provide a reasonable approximation of the resources sector in aggregate. In the Australian context, they are consistent with the fact that the majority of extracted natural resources are exported, that firms engage in both exploration and extraction activity, and that firms import capital and use domestic labour.\(^6\) Although some may view the assumption of a globally competitive resources market as a strong one, we view that this assumption is an appropriate starting point for understanding the mechanism of interest and leave the effects of non-competitive environments and endogenous reserves as an area for future research.\(^7\)

Formally, we assume a continuum of identical resource firms of unit measure. Each period a firm uses capital \((K_t)\), labour \((H^r_t)\) and its existing stock of natural reserves \((R_t)\), to extract a natural commodity \((X_t)\) according to a Cobb-Douglas technology

\[
X_t = (H^r_t A_t)^\eta K_t^\gamma R_t^{1-\eta-\gamma}
\]

\(^5\) For simplicity, we abstract from the use of commodities in domestic production.

\(^6\) See Connolly and Orsmond (2011) for further discussion.

\(^7\) It should be noted that perfect competition is the typically the norm in literature modelling a resources sector within a small open economy.
where $A'_r$ allows for labour-augmenting technical change. There are two additional technical constraints for a resources firm. One is the law of motion for resource-specific capital owned by the firm

$$K_{t+1} = (1 - \delta)K_t + \left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right)I_t$$  \hspace{1cm} (1)$$

where $K_t$ is resource-specific capital, $I_t$ is resource specific investment goods (purchased from abroad), and $\Xi$ is a real convex investment adjustment cost function. The law of motion for resource-specific capital is standard, although we do we allow for adjustment costs on changes in investment (rather than the level of investment relative to the capital stock). This proves to be a convenient reduced form for capturing time to build and lumpiness at the micro-level on project completion. As shown further below, modelling adjustment costs in this way is useful for capturing the typically “hump-shaped” response in investment in response to resource price shocks (see Figure 2).

The second constraint is the law of motion for reserves

$$R_{t+1} = R_t + \omega_{t+1}D_t - \lambda X_t$$  \hspace{1cm} (2)$$

where reserves are depleted through production (extraction) $X_t$ and accumulated through $D_t$, a measure of exploration (or discovery) activity. The parameter $\lambda$ is an indicator variable that turns on or off the effects of resource depletion and is useful when comparing the endogenous and exogenous equilibria defined in the following discussion.

We assume that exploration activity is a uncertain geological process, captured by the random variable $\omega_{t+1}$, which only becomes known at the beginning of period $t+1$. We assume $\omega_{t+1}$ is independently identically distributed on a finite support with distribution function $\Gamma$ and first moment $E(\omega_{t+1}) = 1$ and so the probability that a unit of exploration results in the successful discovery of a unit of new reserves is independent of the state of the economy.

Given the assumption of Cobb-Douglas technology, the total wage bill for a resource firm is given by

$$TC_t^r = \frac{W_t^r}{A_t^r} \left( X_t^{1+\zeta} K_t^{-\mu} R_t^{\mu-\zeta} \right)$$
where \( W' \) is the wage paid to labour used and, following Bohn and Deacon (2000), we define for convenience the parameters \( \zeta = \frac{1}{\eta} - 1 \) and \( \mu = \frac{\gamma}{\eta} \). The firm chooses its investment in resource-specific capital, its extraction, and exploration expenditure by solving the following dynamic program

\[
V(K_t, R_t) = \max_{I_t(j), D_t(j), X_t(j)} \{ S_t P^*_t X_t - \frac{W'_t}{A'_t} \left( X_t^{1+\zeta} K_t^{-\mu} R_t^{\mu - \zeta} \right) - S_t P^*_t I_t - C(D_t, \tilde{R}_t) + \beta \int V(K_{t+1}, R_{t+1}) d\Phi(\xi_{t+1} | \xi_t) \}
\]

where the \( K_{t+1} \) and \( R_{t+1} \) are given constraints (1) and (2) respectively, \( V : \mathbb{R}^2 \to \mathbb{R} \) is the value function, \( S_t \) is the nominal exchange rate, \( \tilde{R}_t \equiv \int_0^1 R_t(j) dj \) is the aggregate stock of domestic reserves, \( P^*_t \) is the price of the extracted commodity in foreign currency terms, \( P^*_t \) is the price of investment goods (imported from abroad) that deliver resource-sector-specific capital in the next period (also measured in foreign currency prices), \( \beta \) is a discount factor, \( \xi_t \) is a state vector containing exogenous prices and aggregate reserves which are known at time \( t \) \( (\xi_t = [P^*_t, W'_t, P^*_t, S_t, R_t, A'_t]) \), and \( C : \mathbb{R}^2 \to \mathbb{R} \) is the cost function associated with exploration activity. We assume that the exploration cost function is convex and that \( Q_r \) units of non-traded goods are required to deliver a unit of discovered reserves. The latter assumption is not important for the partial equilibrium analysis that follows, but is useful when matching the empirical-VAR IRFs.

Uncertainty over future prices, the aggregate stock of reserves, and the success of future exploration are captured in the expectation of the value function in the next period. In the partial equilibrium analysis that follows, we assume that the factor prices and real exchange rate \([W'_t, P^*_t, S_t]\) remain constant in the face of a resource price shock.

---

8 In partial equilibrium we abstract from a stochastic discount factor since firm ownership is not modelled explicitly. This issue will be addressed in the subsequent discussion of the general equilibrium model.

9 See Appendix C for further discussion of the restrictions on the cost function, which imply that it is well defined.

10 Note since \( \phi_{t+1} \) is i.i.d with a unitary first moment, we can in fact integrate this variable out of \( E_t(V(K_{t+1}, R_{t+1})) \).
Our approach is quite similar to that adopted in Bohn and Deacon (2000). However, one important different is that we abstract from the presence of a known finite bound on the cumulative level of resources to be discovered. Consistent with Pindyck (1978), we assume that additional reserves can be discovered in perpetuity but that is increasingly costly to discover new reserves as the level of stock of known reserves increases. This assumption is important for our analysis because it implies that the policy functions that solve the resources firm’s problem are recursive and stationary, and so can be simply integrated within a DSGE model. As well as increasing tractability, we that think this assumption is realistic for many countries, including Australia, given that reserves, production and exploration have continued to grow over time rather than decrease as one would expect in a model of fixed potential reserves (see also Pindyck 1978).\footnote{See Appendix C for further discussion.}

In a symmetric equilibrium, the first-order conditions associated with the resource firms’ problem are given by

\begin{align*}
S_t P_t^{r*} &= (1 + \zeta) \frac{W^r_t}{A^r_t} X^r_t \xi K_t^{-\mu} R_t^{-\zeta} + Q_t' \\
S_t P_t^* &= Q_t^k \left( 1 - \Xi \left( \frac{I_t}{I_{t-1}} \right) - \Xi' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) \\
&\quad + \beta E_t \left( Q_{t+1}^k \Xi' \left( \frac{I_{t+1}}{I_t} \right) \frac{I_{t+1}}{I_t^2} \right) \\
\frac{\partial C(D_t, R_t)}{\partial D_t} &= Q_t' \\
K_{t+1} &= (1 - \delta) K_t + \left( 1 - \Xi \left( \frac{I_t}{I_{t-1}} \right) \right) I_t \\
R_{t+1} &= R_t + \omega_{t+1} D_t - \lambda X_t
\end{align*}

\footnotetext{11 See Appendix C for further discussion.}
where the marginal valuations of an extra unit of reserves and capital to the firm are respectively given by

\[ Q^r_t = \beta E_t \left( (\zeta - \mu) \frac{W^r_{t+1}}{A^r_{t+1}} X^{1+\zeta}_{t+1} K^{\mu-\zeta-1}_{t+1} R^{\mu-\zeta}_{t+1} + Q^r_{t+1} \right) \]  

(8)

\[ Q^k_t = \beta E_t \left( \left( \mu \frac{W^r_{t+1}}{A^r_{t+1}} X^{1+\zeta}_{t+1} K^{\mu-1}_{t+1} R^{\mu-\zeta}_{t+1} + Q^k_{t+1} (1 - \delta) \right) \right) \]  

(9)

Equation (3) implies that firms equate the marginal revenue of extraction with the marginal cost of extraction, where the marginal cost of extraction includes both the additional cost of extraction in period \( t \), and the opportunity cost tied to the fact that resources extracted today cannot be extracted in future periods. Equation (4) implies that the marginal cost of purchasing resource-specific capital from abroad is equal to the marginal return of investing in this capital after accounting for the fact that additional investment reduces future investment-adjustment costs.

Equation (5) implies that the firm will equate the marginal cost of exploration activity with the expected marginal return given by the shadow price of an extra unit of reserves. Equations (6) and (7) describe the law of motion for capital and the stock of natural reserves respectively. The shadow prices in Equations (8) and (9) reflect the marginal valuations of an additional unit of reserves and an additional unit of capital respectively, and are given by the present discounted value of the additional revenue streams generated by either an extra unit of reserves or capital.

We compare two equilibria associated with these first-order conditions. The first is the model where resources are depletable (\( \lambda = 1 \)) and exploration expenditure responds to changes in prices.

**Definition 1.** A partial equilibrium for the endogenous reserves model is given by sequences for \( \{X_t, D_t, I_t, K_{t+1}, R_{t+1}, q^R_t, q^K_t\} \) that solves Equations (3) to (9) taking the expected sequences \( \{W^m_t, S_t, P^*_t, P^{r*}_t, A^r_t\} \) as given and assuming \( \lambda = 1 \).

The second equilibrium we consider assumes that the stock of resources is exogenous, and thus abstracts from both depletion and the scope for exploration activity.\(^{12}\)

\(^{12}\) It is straightforward to verify that the steady states for these equilibria exist and are identical.
Definition 2. A partial equilibrium with exogenous reserves, is given by sequences \( \{X_t, I_t, K_{t+1}, R_{t+1}, q_t^R, q_t^K\} \) that solves Equations (3) to (4) and (6) to (9) taking the expected sequences \( \{W_m^t, S_t, P^*_{t}, P^*_{R}, \omega_{t+1}, A_t^r\} \) as given and assuming \( \lambda = 0 \) and that \( D_t = 0 \) for all \( t \).

3.2 Partial Equilibrium Calibration

Table 1 reports the calibration for the structural parameters with the model solved at an annual decision horizon – the highest frequency with which production and reserves data are available. The discount factor and depreciation rate are chosen to be in line with existing literature that models a resources sector.\(^{13}\) For the exponents on capital and labour in the resource extraction technology (\( \gamma \) and \( \eta \)), these are chosen to match a steady state rate of annual extraction of two per cent, and a wage bill relative to total revenue of approximately 11 per cent.\(^{14}\) The two percent average annual extraction rate is consistent with an equally-weighted average of extraction rates in iron-ore, coal, gold, lead, nickel, zinc, copper and bauxite for the sample 1976 to 2011.\(^{15}\)

For the parameterisation of exploration costs, we use a function that implies that resource sector profits are linearly homogenous\(^{16}\)

\[
C \left( D_t, \tilde{R}_t \right) = P_t^n Q^r \phi_{mc} \left( \frac{D_t}{R_t} - \frac{D_{t-1}}{R_{t-1}} \right) \tilde{R}_t
\]

where \( \phi_{mc} \) is a parameter that governs the sensitivity of exploration costs to shocks – and, thus, the incentive to engage in exploration activity – \( Q^r \) is a normalisation

\(^{13}\) See for example Charnavoki (2010) and Garcia and Gonzalez (2010).

\(^{14}\) This estimate is consistent with estimates from Topp, Soames, Parham and Bloch (2008) and ABS catalogue 8414.0.

\(^{15}\) More specifically, we use an arithmetic average across these industries using both the extraction weights implied when using economically demonstrated reserves (2.8 per cent per annum) and total reserves (1.65 percent per annum) where the latter also include sub- and para-marginal reserves.

\(^{16}\) That is, a doubling of production and of all factor inputs, including reserves, would double revenue and double cost.
used to ensure a well defined steady state, and $P^n_t$ is the price of a bundle of non-traded goods (held fixed for the partial equilibrium analysis).

Importantly, and as discussed further in Appendix C, this cost function satisfies the restrictions that exploration costs are increasing in both exploration and aggregate reserves, $\frac{\partial C}{\partial D_t} > 0, \frac{\partial C}{\partial \tilde{R}_t} > 0$, that the derivative of the marginal cost of exploration is increasing in the level of exploration, $\frac{\partial^2 C}{\partial D^2_t} > 0$, and that this latter derivative is sufficiently large that it outweighs any reduction in the marginal costs of exploration that are tied to greater existing reserves permitting further delineation or extensions of existing deposits $\frac{\partial^2 C}{\partial D_t^2} + \frac{\partial^2 C}{\partial D_t \partial \tilde{R}_t} > 0$.

For the parameterization of investment adjustment costs we assume a quadratic adjustment cost function satisfying $\Xi'(1) = 0, \Xi''(1) = \kappa$. For resource prices we assume that the natural log of prices follow a simple AR(1) process with autoregressive parameter $\rho_r$.

$$\ln P^r_t = \rho_r \ln P^r_{t-1} + \epsilon^r_t$$

where $\epsilon^r_t$ is independently identically distributed (i.i.d). All other prices $\{W^r_t, S_t, P^*_t\}$ are held fixed in partial equilibrium.

The parameters $\phi_{mc}, \kappa$ and $\rho_r$ only affect the dynamics of impulse response functions (IRFs) and have no bearing on the steady state of the model in partial equilibrium. For this reason, the values for all three are chosen to be consistent with the values implied when matching the IRFs of the general equilibrium version of our model (discussed further below) with the empirical VAR previously discussed.

### 3.3 Results

Figures 3 and 4 highlight the impulse response functions associated with a one per cent positive innovation in resource prices, and holding fixed wages, the exchange rate and the price of imported capital. The impulses are computed under both the endogenous and exogenous reserves equilibria. Comparing these two equilibria, it is clear that the endogenous model generates greater amplification and persistence in response to a resource price shock. Factor utilisation for both labour and capital increase by more in the endogenous reserves model, as does the level of extraction.
Table 1: Resources Sector Parameterisation

<table>
<thead>
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<th>Calibrated Parameters</th>
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<tbody>
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</tr>
<tr>
<td>Labour factor exponent</td>
<td>γ</td>
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<td>0.10</td>
</tr>
<tr>
<td>Depreciation rate</td>
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</tr>
</tbody>
</table>

Parameters obtained from GMM Estimation of General Equilibrium Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploration costs dynamics  ( \phi_{mc} )</td>
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</tr>
<tr>
<td>Investment cost parameter ( \kappa )</td>
<td>3</td>
</tr>
<tr>
<td>AR(1) parameter (prices) ( \rho_r )</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The stock of reserves also increases in the endogenous reserves model (held constant by assumption with exogenous reserves) as exploration activity and the discovery of new deposits results in reserves accumulating faster than they can be depleted through extraction.

The mechanism driving the amplification of the resources price shock is the feedback effects that occur through exploration. As a persistently higher price provides firms with an incentive to engage in the exploration of new reserves, or to find extensions to existing deposits, the expected value of newly discovered reserves increases leading firms to engage in exploration activity. Importantly, any newly discovered reserves are a permanent addition to resources firm production opportunity set. That is, once discovered, they can be extracted either in the next period or in any future period without depreciation. It is this permanent increase in the stock of reserves that generates the magnitude and persistence of the responses in the endogenous reserves model.

Formally, allowing for endogenous reserves results in an endogenous unit root in the equilibrium law of motion for reserves.\(^\text{17}\) That is, transitory changes in resource prices can generate permanent changes in investment, labour utilisation and resource sector production. We use the word endogenous unit root in this context to highlight that reserve dynamics are a function of resource prices.\(^\text{18}\)

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\(^\text{17}\) In Appendix A we discuss two cases in which this result can be demonstrated analytically. The analytical results highlight the importance of the assumptions of perfect competition, constants returns to scale in the extraction technology, linearly homogenous resource sector profits, and the exogeneity of resource prices. Together, these assumptions imply that in equilibrium reserves will be non-stationary and an endogenous function of resource prices.
This result can help to explain why the scale of the resources sector appears to trend over time, even if real resource prices exhibit long-run mean reversion (stationarity) as we have assumed.

In view of the additional amplification generated in response to a resources sector price shock, we now investigate whether the same results hold in general equilibrium, and whether the incorporation of endogenous reserves changes the broader effects of a resource price shock on the rest of the economy.

**Figure 3: Response to a 1 per cent Increase in Resource Prices in Partial Equilibrium**

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18 This is in contrast to literature modelling exogenous sectoral unit roots, where different trend growth rates in sectors are assumed rather than derived as endogenous feature of the modelling framework.
Figure 4: Response to a 1 per cent Increase in Resource Prices in Partial Equilibrium (Continued)
4. Modelling Natural Resources in General Equilibrium

In general equilibrium, we assume that the resources sector is largely identical to that previously discussed,

\[ S_t p^*_t = (1 + \zeta) \frac{W_t}{A_t} X_t^\gamma K_t^{-\mu} R_t^{\mu - \zeta} + Q_t^r \]  

(10)

\[ S_t p^*_t = Q_t^k \left( 1 - \Xi \left( \frac{I_t}{I_{t-1}} \right) \right) - \Xi' \left( \frac{I_t}{I_{t-1}} \right) \]  

+ \beta E_t \left( M_{t,t+1} Q_{t+1}^k \Xi' \left( \frac{I_{t+1}}{I_t} \right) \frac{I_{t+1}^2}{I_t^2} \right) \]  

(11)

\[ \frac{\partial C(D_t, R_t)}{\partial D_t} = Q_t^r \]  

(12)

\[ K_{t+1} = (1 - \delta) K_t + \left( 1 - \Xi \left( \frac{I_t}{I_{t-1}} \right) \right) I_t \]  

(13)

\[ R_{t+1} = R_t + \omega_{t+1} D_t - \lambda X_t \]  

(14)

\[ Q_t^r = \beta E_t \left( M_{t,t+1} \left( \zeta - \mu \right) \frac{W_{t+1}}{A_{t+1}} X_{t+1}^{1+\xi} K_{t+1}^{-\mu} R_{t+1}^{\mu - \zeta} + Q_{t+1}^r \right) \]  

(15)

\[ Q_t^k = E_t \left( M_{t,t+1} \left( \mu X_{t+1}^{1+\xi} K_{t+1}^{-\mu} R_{t+1}^{\mu - \zeta} + Q_{t+1}^k \right) \right) \]  

(16)

\[ \Psi_t^R = S_t p^*_t X_t - \frac{W_t}{A_t} \left( X_t^{1+\xi} K_t^{-\mu} R_t^{\mu - \zeta} \right) - S_t p^*_t I_t - C(D_t, R_t) \]  

(17)

\[ X_t = (H_t^F \gamma) K_t^{\gamma} R_t^{1-\gamma} \]  

(18)

However, to properly integrate a resources firm problem within general equilibrium we require two further assumptions. The first assumption is that we now explicitly account for the preferences of resource firm owners. This is done by assuming that firms use a stochastic discount factor (SDF), \( \beta M_{t,t+1} \), when valuing profits over time and states of the world rather than the deterministic discount factor, \( \beta \). For simplicity, and consistent with the presence of foreign ownership in the sector, we assume that the SDF for resources firms only partially updates to
reflect the preferences of the domestic firm owners and is thus given by

\[ M_{t,t+1} = \left( \nu \frac{\Theta_{t+1}}{\Theta_t} + 1 - \nu \right) \frac{P_t^c}{P_{t+1}^c} \]

where \( \nu \) is the parameter governing the importance of domestic ownership and \( \Theta_t \) is the marginal utility of (domestic household) consumption in period \( t \).

The second assumption used is to explicitly model inputs into exploration activity. Consistent with the previous parameterization of exploration costs, we assume that resources firms purchase a bundle of non-traded goods using the Dixit-Stiglitz aggregator

\[ Y_t^{r,n} = \left( \int_0^1 Y_t^{r,n} \frac{\theta_{n-1}}{\theta_n} \theta_{n-1} \theta_n di \right) \]

and that this bundled good is then used as an input in the discovery process

\[ D_t = \frac{Y_t^{r,n}}{Q'} \phi_{mc} \]  

(19)

The additional convexity in the cost function is introduced through the term \( e^{\phi_{mc} \left( \frac{D_t}{R_t} - \frac{D}{R} \right) R_t}, which captures the effect of resource scarcity. This term implies that increasing the ratio of exploration, \( D_t \), to reserves, \( R_t \), becomes increasingly expensive as this ratio rises.

We now discuss the rest of the small open economy. Our approach is very similar to that employed in Adolfson, Lasen, Lind and Villani (2007) and so we aim to capture some basic empirical regularities consistent with a small open economy, including non-tradeable goods, incomplete pass through, and a risk premium that affects the ability of domestic residents to borrow from abroad (the latter also ensuring a stationary model solution as discussed in Schmitt-Grohe and Uribe (2003)). However, to keep the analysis tractable, we abstract from many of the frictions included in the model of Adolfson et al (2007). For example, we choose not to include wage rigidities, variable capital utilisation or financing frictions. Although these mechanisms could be important for the dynamics of the model quantitatively, we do not expect that the qualitative implications discussed
below would be fundamentally different in the presence of these additional rigidities.

4.1 Domestic Households

We assume a continuum of identical domestic households of unit measure who are able to self-insure with each other, and so the problem we describe is isomorphic to a model with a representative agent. Each household has identical preferences given by the utility function

\[
U_{t_0} = E_{t_0} \sum_{t = t_0}^{\infty} \beta^{t-t_0} \left( \frac{(C_t)^{1-\xi_c}}{V_t^{1-\xi_c}} + \varsigma \left( H_t^{n_h^{\gamma_h}} + H_t^{r_h^{\gamma_h}} \right)^{\xi_h \gamma_h} \right)
\]

where \(C_t\) is an aggregate consumption bundle containing domestic and imported goods, \(V_t\) is an external habit, \(H_t^n\) and \(H_t^r\) are the households’ supply of labour to the non-resource and resource sectors respectively, \(\xi_c\) is the coefficient of relative risk aversion, \(\xi_h\) is a parameter governing the convexity of preferences with regard to the aggregate supply of labour, \(\gamma_h\) governs the elasticity of substitution between labour supplied in the resources and non-resources sectors, and \(\varsigma\) is a parameter used to obtain a well-defined steady state.

We include an external habit or “catching up with the Jones” \(V_t = \int_0^1 C_{t-1}(j) \, dj\), see Abel 1990) because it permits a more flexible representation of consumption preferences. In particular, it allows for a non-unitary coefficient of relative risk aversion while still being consistent with a detrended stationary representation of the general equilibrium economy. We use a constant-elasticity-of-substitution function for the dis-utility of labour to capture the idea that working in the resources and non-resources sector are not perfect substitutes, from the perspective of households, and so there can be relative wage dispersion between the resources and non-resources sectors.

Regarding financial markets, domestic households can trade in either of two nominal risk-less bonds denominated in the domestic, \(B_t\), and foreign currencies, \(B_t^*\), respectively. Following Benigno and Thoenissen (2008), we assume that when domestic residents issue claims in foreign currency they must pay a premium
on this borrowing, $\Phi_t$. The flow household budget constraint for an individual household is given by

$$P^C_t C_t + \frac{B_t}{(1 + i_t)} + \frac{S_t B_t^*}{(1 + i_t^*)} = W^n_t H^n_t + W^f_t H^f_t + B_{t-1} + S_t B_{t-1}^* + \nu \Psi^f_t + \Psi^n_t + \Psi^o_t + \Psi^x_t - T$$

where $\Psi^r_t, \Psi^n_t, \Psi^o_t, \Psi^x_t$ are the aggregate profits (dividends) of the resource, non-resource, import and non-resource export goods sectors paid to household respectively, $T_t$ is a lump-sum tax used to fund subsidies that undo the steady state distortions associated with monopolistic competition (discussed further below), and $i_t$ and $i_t^*$ are the domestic and foreign borrowing interest rates (the latter being measured net of the risk premium). Assuming that domestic bonds are in zero net supply, it follows that in a symmetric equilibrium

$$\Theta_t = \frac{1}{C_t} \left( \frac{C_t}{V_t} \right)^{1-\xi_c}$$

$$\Theta_t = \beta E_t \left( (1 + i_t) \frac{P^c_t}{P^c_{t+1}} \Theta_{t+1} \right)$$

$$\Theta_t = \beta E_t \left( (1 + i_t^*) \Phi_t \frac{P^c_t}{P^c_{t+1}} \Theta_{t+1} \frac{S_{t+1}}{S_t} \right)$$

$$\frac{W^n_t}{P^c_t} \Theta_t = \zeta H^n_t \gamma^{-1}_{t} \left( H^n_t \gamma^{-1}_{t} + H^f_t \gamma^{-1}_{t} \right)$$

$$\frac{W^f_t}{P^c_t} \Theta_t = \zeta H^f_t \gamma^{-1}_{t} \left( H^n_t \gamma^{-1}_{t} + H^f_t \gamma^{-1}_{t} \right)$$

$$P^c_t C_t = W^n_t H^n_t + W^f_t H^f_t + S_t B_{t-1}^* - \frac{S_t B_{t-1}^*}{(1 + i_t^*)} \Phi_t + \nu \Psi^f_t + \Psi^n_t + \Psi^o_t + \Psi^x_t - T$$

The first two equations are the standard Euler equations associated with ability to trade in domestic and foreign currency denominated bonds and imply that the return from saving (or cost of borrowing) should be equal to the forgone (additional) consumption enjoyed in the current period. Equations (24) and (25)
are the standard intra-temporal conditions ensuring that the marginal return from working in each sector is equivalent to the households’ marginal disutility from working, and Equation (26) implies that the household budget constraint will bind.

We assume that the risk-premium on foreign borrowing is parameterised by the following relationship

\[ \Phi_t \equiv \exp \left( \left( \frac{S_t B_t^*}{P_t^c R_t} - \frac{S B^*}{P^c R} \right)^{-\varphi_b} \left( \ln \frac{S_{t-1}}{P_{t-1}^c} - \ln \frac{P_{t-1}^f}{P^f} \right)^{\varphi_s} \right) \]

That is, we assume that the risk premium is a function of both the domestic economy’s capacity to repay its debt, and the percentage deviation between the real exchange rate and resource prices. We include the latter term to capture the idea that the combination of high resource prices and an appreciated real exchange rate can imply that international investors will view the country as a relatively favourable investment destination, and require less compensation for risk. The parameters \( \xi \) and \( \varphi \) govern the relative importance of each of these effects and are estimated.

For the intra-temporal consumption allocations to domestic and imported consumption good bundles, each household solves

\[
\min P_t^p C_t^p + P_t^o C_t^o \\
\text{subject to:} \\
C_t \leq \left[ (1 - \alpha) \frac{1}{\tilde{\eta}_c} \left( C_t^p \right)^{\frac{\eta_c - 1}{\eta_c}} + (\alpha) \frac{1}{\tilde{\eta}_c} \left( C_t^o \right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}}
\]

where \( P_t^p \) and \( P_t^o \) are the prices of the non-traded and imported goods purchased respectively. Defining the shadow price of the aggregate consumption bundle as
$P^c_t$, the optimality conditions are given by

\[
C_t = \left[ (1 - \alpha)^{\frac{1}{\eta_c}} (C^n_t)^{\frac{\eta_c - 1}{\eta_c}} + (\alpha)^{\frac{1}{\eta_c}} (C^\alpha_t)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{1}{\eta_c - 1}} \tag{27}
\]

\[
C^n_t = (1 - \alpha)^{-\eta_c} C_t \tag{28}
\]

\[
C^\alpha_t = (P^\alpha_t)^{-\eta_c} C_t \tag{29}
\]

where the shadow price of consuming an additional bundle of non-traded and imported goods is given by

\[
P^c_t = \left[ (1 - \alpha) \left( P^n_t \right)^{1 - \eta_c} + \alpha \left( P^\alpha_t \right)^{1 - \eta_c} \right]^{\frac{1}{1 - \eta_c}}
\]

To find the consumption allocations within the non-traded goods bundle, a household solves

\[
\min \int_0^1 P^n_{it} C^n_{it} \, di
\]

subject to:

\[
C^n_t \leq \left( \int_0^1 C^n_{it} \frac{\theta_n}{\eta_n} \, di \right)^{\frac{\theta_n}{\eta_n - 1}}
\]

From which the shadow price and consumption allocations are given by

\[
P^n_t = \left( \int_0^1 P^n_{it}^{1 - \theta_n} \, di \right)^{\frac{1}{1 - \theta_n}}
\]

\[
C^n_{it} = \left( \frac{P^n_{it}}{P^n_t} \right)^{-\theta_n} C^n_t
\]

\[
C^n_t = \left( \int_0^1 C^n_{it} \frac{\theta_n - 1}{\eta_n} \, di \right)^{\frac{\theta_n}{\eta_n - 1}}
\]
Solving the analogous problem for the imported goods consumption allocations we have

\[ P_t^o = \left( \int_0^1 P_{it}^{o_1 - \theta_o} di \right)^{\frac{1}{1 - \theta_o}} \]

\[ C_{it}^o = \left( \frac{P_{it}^o}{P^o_t} \right)^{-\theta_o} C^o_t \]

\[ C^o_t = \left( \int_0^1 C_{it}^{o_1 - \theta_o} di \right)^{\frac{\theta_o}{1 - \theta_o}} \]

### 4.2 Domestic Non-tradeable Producers

We assume a continuum of non-tradeable consumption producers on the unit interval. Each non-tradeable producer, indexed by \( i \), has access to a linear production technology

\[ Y_{it}^{s,n} = \frac{A^n_t}{\chi_t} H^n_{it} \]

where \( A^n_t \) is a common non-traded technology, \( H^n_{it} \) is the quantity of non-traded labour used by firm \( i \), and \( \chi_t \) can be interpreted as a cost-push shock. We include the later to allow for the idea that energy prices are typically correlated with resource prices, and so in principle a resources price shock could have a cost-push dimension. That is, firms have to pay more for the energy they use when resource prices rise.\(^{19}\) We assume that \( \chi_t \) follows has the same autoregressive (AR1) process as that modelled for resource prices, but that the covariance between firms energy costs and resources price is a free parameter to be estimated \((\Upsilon)\)

\[ \ln \chi_t = \rho_r \ln \chi_{t-1} + \Upsilon \epsilon_t^{r,s} \quad (30) \]

For competitive structure, we assume non-traded firms operate under monopolistic competition and are subject to a Calvo pricing friction. For the fraction \((1 - \phi_n)\)

---

\(^{19}\) Note that this assumption is consistent with the use of commodity prices as control for expected inflation in VARs that attempt to identify the effects of monetary policy shocks and address the so called “prices puzzle” (see for example Sims 1992). It is also consistent with the observed correlation between commodity prices and inflation rates across countries that cannot be explained by correlation in real activity (see for example Gerard 2012).
of firms able to set their price optimally, they solve

$$\max_{P_i^n} \sum_{t=t_0}^{\infty} \left( \phi_n \beta \right)^{t-t_0} \frac{P_{i_0}^c \Theta_t}{P_t^c \Theta_{t_0}} \left( \frac{P_{i_0}^n - (1 - \tau_n) MC_{i_0}^n}{P_t^n} \right)^{-\theta_n} Y_{d,n}$$

where

$$MC_{i_0}^n = \frac{W_{i_0}^n}{A_i^n} \chi_t$$

is the marginal cost of production for a domestic non-traded producer, $Y_{d,n}$ is a measure of common (non-idiosyncratic) non-traded demand and $\tau_n$ is a subsidy used to undo the steady state distortion associated with the assumption of monopolistic competition (recall this is funded by the lump-sum tax on households). A recursive formulation of the implied optimality conditions is

$$P_t^n = (1 - \tau_n) \left( \frac{\theta_n}{\theta_n - 1} \right) \frac{V_t^n}{U_t^n} \tag{31}$$

$$V_t^n = Y_{d,n} \Theta_t \left( \frac{P_t^n}{P_t^c} \right)^{\theta_n} W_t^n A_t^n \chi_t + \beta \phi_n E_t \left( V_{t+1}^n \right) \tag{32}$$

$$U_t^n = Y_{d,n} \Theta_t \left( \frac{P_t^n}{P_t^c} \right)^{\theta_n} + \beta \phi_n E_t \left( U_{t+1}^n \right) \tag{33}$$

where $P_t^n$ is the optimal reset price for the firm. It should be noted that in equilibrium all firms will choose the same optimal reset price given that there will be a degenerate wage distribution in an equilibrium where all households are identical ($W_{i_0}^n = W_t^n$) and that there are no idiosyncratic shocks. For the remaining fraction ($\phi_n$) of firms not able to choose their price, they simply retain the price they offered in the previous period. Accordingly, a measure of non-traded goods prices, the shadow price of an extra bundle of non-traded consumption goods, is

$$P_t^n = \left( (1 - \phi_n) \left( P_t^n \right)^{1-\theta_n} + \phi_n P_{t-1}^{n-\theta_n} \right)^{\frac{1}{1-\theta_n}} \tag{34}$$

It is straightforward to verify that the total profit of domestically owned non-traded producers is given by

$$\int_0^1 \Psi^n_{it} di = P_t^n Y_{d,n} - (1 - \tau_n) W_t^n H_t^n \tag{35}$$
4.3 Domestic Importing Firms

We assume a continuum of importing firms of unit measure who are owned by domestic households. Importing firms purchase final output from the foreign sector at the foreign currency price, \( P_t^* \), and use this output to produce a differentiated imported good. The real marginal cost common to all importers in domestic currency terms is

\[ MC^o_t = S_t P_t^* \]

Assuming that importers operate under monopolistic competition, and that a Calvo pricing friction exists for importers resetting their domestic currency price, we have

\[
\bar{P}_t^o = (1 - \tau^o) \left( \frac{\theta^o}{\theta^o - 1} \right) \frac{V_t^o}{U_t^o} \tag{36}
\]

\[
V_t^o = C_t^o \Theta_t \left( \frac{P_t^o}{P_t^c} \right)^{\theta^o} S_t P_t^* + \beta \phi^o E_t (V_{t+1}^o) \tag{37}
\]

\[
U_t^o = C_t^o \Theta_t \left( \frac{P_t^o}{P_t^c} \right)^{\theta^o} + \beta \phi^o E_t (U_{t+1}^o) \tag{38}
\]

where \( \bar{P}_t^o \) is the optimal reset price chosen by importers able to choose their price, and \( \phi^o \) is the probability that any given firm will not be able to re-optimise its price in a given period and retains its previous period price. The shadow price relevant for imported goods is

\[
P_t^o = \left( 1 - \phi^o \right) \left( \bar{P}_t^o \right)^{1-\theta^o} + \phi^o P_{t-1}^{o\theta^o} \right)^{\frac{1}{1-\theta^o}} \tag{39}
\]

For determining import-firm profits we define the alternative import price index

\[
\tilde{P}_t^o = \left[ \int_0^1 P_t^o (i)^{-\theta^o} di \right]^{-\frac{1}{\theta^o}} \tag{40}
\]

\[
= \left( 1 - \phi^o \right) \left( \bar{P}_t^o \right)^{-\theta^o} + \phi^o \bar{P}_{t-1}^{o\theta^o} \right)^{\frac{1}{\theta^o}}
\]
where total profit in the imported sector is given by

\[ \int_0^1 \Psi_{it}^o di = P_t^o C_t^o - (1 - \tau_o) S_t P_t^* \left( \frac{\tilde{P}_t^o}{P_t^o} \right)^{-\theta_o} C_t^o \]  

(41)

### 4.4 Non-Resource Export Sector

Given substantial interest in how resource sector developments can influence the non-resource export sector, we also assume a unit measure of domestically owned firms that engage in non-resource exporting (hereafter, exporters). An exporter, indexed by \( j \), purchases a bundle of non-traded inputs from domestic producers and transforms this into a specialised export good. The demand for inputs from non-traded producer \( i \) by exporter \( j \) is given by

\[ Y_{it}^{n,x}(j) = \left( \frac{P_{it}^n}{P_t^n} \right)^{-\theta_n} C_t^x(j) \]

where \( C_t^x(j) \) is the demand for exporter \( j \)'s output. The real marginal cost for an exporter is

\[ MC_t^x(j) = P_t^n \]

We assume the following (reduced-form) demand function for exports of type \( j \)

\[ C_t^x(j) = \left( \frac{P_{jt}^{x*}}{P_t^{x*}} \right)^{-\theta_x} C_t^* \]

\[ C_t^* = \left( \frac{P_t^*}{P_t} \right)^{-\theta_*} Y_t^* \]

where \( C_t^* \) is a measure of the common component of demand for non-resource exports, \( Y_t^* \) is foreign output, \( P_{jt}^{x*} \) is the price of export type \( j \) in foreign currency terms, \( P_t^{x*} \) is an index of non-resource export prices in foreign currency terms, and \( P_t^* \) is the foreign price index. Note that \( \theta_x \) is the within sector elasticity of non-resource export demand, and that \( \theta_* \) is the cross-sector (or common) elasticity of non-resource export demand. Consistent with Adolfson et al (2007), this formulation allows for both competition effects amongst firms within the
exporting sector, and competition between the export sector as a whole and the rest of the world.

Assuming that exporters are monopolistically competitive, set their prices in foreign currency terms, and are subject to a Calvo pricing friction, a recursive formulation for determining their optimal reset price is

$$P_x^* = (1 - \tau_x) \left( \frac{\theta_x}{\theta_x - 1} \right) \frac{V_t^x}{U_t^x} \quad (42)$$

$$V_t^x = C_t^* \Theta_t \left( P_t^{x^*} \right)^{\theta_x} P_t^n \frac{P_t^n}{P_t^c} + \beta \phi_x E_t (V_{t+1}^x) \quad (43)$$

$$U_t^x = C_t^* \Theta_t \left( P_t^{x^*} \right)^{\theta_x} \frac{S_t}{P_t^c} + \beta \phi_x E_t (U_{t+1}^x) \quad (44)$$

where the price index for non-resource exported goods is defined by

$$P_t^{x^*} = \left[ \int_0^1 P_{jt}^{x^* 1-\theta_x} dj \right]^{\frac{1}{1-\theta_x}}$$

$$= \left( \left( 1 - \phi_x \right) \left( P_t^{x^*} \right)^{1-\theta_x} + \phi_x \left( P_{t-1}^{x^*} \right)^{1-\theta_x} \right)^{\frac{1}{1-\theta_x}} \quad (45)$$

It will be useful for determining non-resource export firm profits to also define the alternative non-resource export price index

$$\tilde{P}_t^{x^*} = \left[ \int_0^1 P_{jt}^{x^* -\theta_x} dj \right]^{-\frac{1}{\theta_x}}$$

$$= \left( \left( 1 - \phi_x \right) \left( P_t^{x^*} \right)^{-\theta_x} + \phi_x \left( \tilde{P}_{t-1}^{x^*} \right)^{-\theta_x} \right)^{-\frac{1}{\theta_x}} \quad (46)$$

Total profit of exporters is thus

$$\int_0^1 \Psi^{x^*}_j dj = S_t P_t^{x^*} C_t^* - (1 - \tau_x) P_t^n \left( \frac{\tilde{P}_t^x}{P_t^x} \right)^{-\theta_x} C_t^* \quad (47)$$
4.5 Monetary Policy

For domestic monetary policy we assume that the central bank follows a simple Taylor rule of the form

$$\ln(1+i_t) = (1-\rho_i)\ln(1+\tilde{i}) + (1-\rho_i)\rho\pi E_t \ln\left(\frac{P_{t+1}}{P_t}\right) + \rho_i \ln(1+i_{t-1}) \quad (48)$$

This rule is consistent with a forward-looking central bank that targets inflation, but also allows for gradual interest rate adjustment.

4.6 Market Clearing and the Rest of the World

For market clearing, supply must meet the demand for each non-traded good $i$

$$Y_{it}^{s,n} = Y_{it}^{d,n}$$

Aggregating demand and supply across the continuum of goods, it follows that

$$\frac{A^n_t H^n_t}{\chi_t} = \left(\frac{\tilde{P}^n_t}{P^n_t}\right)^{-\theta_n} Y_{t}^{d,n} \quad (49)$$

and where

$$Y_{t}^{d,n} \equiv \int_0^1 (C^n_t + C^x_t + Y_{t}^{r,n}) \, dj - D\frac{Q^r}{\phi_{mc}} \quad (50)$$

$$\tilde{P}^n_t \equiv \left[\int_0^1 P^n_{it}^{-\theta_n} \, di\right]^{-\frac{1}{\theta_n}} \quad (51)$$

Note that the common or non-idiosyncratic component of demand for non-traded goods is given by the sum of demand for non-traded consumption goods, demand for non-traded inputs which are then exported, and net demand for non-traded inputs used up in the exploration process.\(^{20}\)

\(^{20}\) For simplicity, and to simplify calculation of the steady state, we assume that the government makes a constant lump-sum allocation of exploration inputs, $D_0\phi_{mc}$, to the resources sector, and so only demand in excess of this steady state allocation actually absorbs production from the non-traded sector.
For the rest of the world, we assume that prices and quantities admit the following Vector Autoregression (VAR($p$)) representation

$$y_t^* = \sum_{j=0}^{p} A_j y_{t-j} + \varepsilon_t \quad (52)$$

where $y_t^* = \begin{bmatrix} Y_t^* & i_t^* & P_t^* & P_t^* \end{bmatrix}'$ is a vector collecting all foreign prices and quantities and $\varepsilon_t$ is a $4 \times 1$ vector of reduced form shocks.\(^{21}\)

**Definition 3.** A small open economy general equilibrium with endogenous reserves, and under rational expectations, is given by sequences of quantities ${\{C_t, C^n_t, Y^d_t, H_t, B_t^*, R_t, D_t, X_t, I_t, K_t, \psi^n_t, \psi^*_t, \psi^r_t, \chi_t, Y^r_t, n_t\}}$ and prices ${\{P^c_t, P^n_t, P^p_t, P^x_t, W_t, W^r_t, S_t, Q^k_t, Q^n_t, Q^r_t, Q^p_t, Q^x_t, \tilde{P}_t, \tilde{P}^n_t, \tilde{P}^p_t, \tilde{P}^x_t, \tilde{P}^r_t, \tilde{P}^c_t, y_t^*, \psi_t^*, \psi_t^r, \psi_t^n, \psi_t^p, \psi_t^x, \psi_t^r, \varepsilon_t, \psi_t^n, \psi_t^p, \psi_t^x, \psi_t^r, \chi_t, Y_t^r, n_t\}}$ that solve Equations (10) to (19) and (21) to (51) taking foreign quantities and prices $\{y_t^*\}$ as given by Equation (52).

In view of the fact that the stock of natural reserves is potentially non-stationary, as highlighted in partial equilibrium, we still need to find a stationary representation of the above economy. The next proposition establishes two assumptions that enable a straightforward calculation of a detrended stationary representation of the above economy

**Proposition 1.** A detrended stationary representation of the above economy exists if

1. Foreign demand and the stock of domestic natural reserves are cointegrated (share the same long-run relationship)

$$\ln \frac{Y_t^*/Y^*}{R_t/R} = \vartheta \ln \frac{Y_{t-1}^*/Y^*}{R_{t-1}/R} - \Delta \ln R_t + \varepsilon_t^*$$

---

\(^{21}\) Although we could alternatively follow a structural specification for the rest of the world, the analysis becomes less transparent when attempting to disentangle the effects of structural shocks that move all foreign prices and quantities simultaneously. Since we are interested in the effect of a change in resource prices, ceteris paribus, we simply assess the marginal effect of this change without ascribing its source to a particular foreign structural shock.
2. Resources sector and non-resources sector technology are also identically cointegrated with the stock of domestic natural reserves

\[
\ln \frac{A^n_t}{R_t/R} = \vartheta \ln \frac{A^n_{t-1}}{R_{t-1}/R} - \Delta \ln R_t + \epsilon^*_t
\]

\[
A^n_t = A^r_t
\]

The existence result underlying this proposition is discussed in Appendix B. Essentially, a simple detrended representation of the economy can be obtained if the relative sizes of the domestic and foreign economy do not exhibit permanent secular trends, as implied by the cointegration between foreign demand and reserves. And similarly, that the relative sizes of the domestic resource and non-resource sectors cannot exhibit permanent secular trends (ensured by the cointegration of the two technology processes). To minimise the effects of these assumptions we assume \( \vartheta = 0.999 \), and so the variation induced by endogenous reserves on \( A^n_t, A^r_t \) and \( Y^*_t \) through cointegration is minimal.²²

4.7 General Equilibrium Results

Calibration

Most parameters in our general equilibrium model are calibrated. For the calibrated resource sector parameters, we use the same calibration previously described in our partial equilibrium analysis (see Table 1). For the rest of the domestic economy, our calibrated parameters are chosen to be in line with the results in Jääskelä and Nimark (2008). These authors estimate a model with a relatively similar production structure to ours and Adolfson et al (2007) using Australian data but with a simple reduced-form for the resources/commodities sector.²³

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²² In effect the high degree of persistence implies that it takes several thousand years for these process to converge to their new long-run equilibrium values.

²³ The main exception is for the calibration of the elasticity of substitution on imported goods. The estimate implied in Jääskelä and Nimark (2008) implies a very large mark-up on imported goods. We abstract from concern over whether this parameter is well identified and simply fix the implied mark-ups on domestically produced and imported goods to be identical.
The parameters we choose are adjusted to match an annual time horizon and are summarised in Table 2. We assume identical elasticities of substitution within the non-traded goods, importing and non-resource export sectors, each consistent with a mark-up of approximately 17 per cent. We further assume identical price stickiness parameters, each implying a 20 per cent probability that a firm will not be able to re-optimise its price within a year’s time.

We choose the home bias parameter to match a 20 per cent import share in steady state, and an elasticity of substitution between consumption of non-traded goods and imports that is close to one (Cobb-Douglas consumption preferences). We set the elasticity of substitution between resources and non-resources labour at 1, and fix the overall convexity parameter of labour disutility at 4. These assumptions imply that labour is relatively substitutable between sectors, but that households are averse to increasing their overall supply of labour to the economy. Finally, we calibrate the weight on the Taylor rule in inflation (prior to estimation of the importance of interest rate smoothing) at 5. This value implies that the central bank is averse to deviations of inflation from target at an annual horizon.  

<table>
<thead>
<tr>
<th>Table 2: Calibration of Non-Resource Economy</th>
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</thead>
<tbody>
<tr>
<td>Household discount factor</td>
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<tr>
<td>Labour convexity</td>
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<tr>
<td>Labour substitution parameter</td>
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<tr>
<td>Consumption substitution elasticity</td>
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<tr>
<td>Home-bias coefficient</td>
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<tr>
<td>Substitution elasticity (within non-traded goods)</td>
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<tr>
<td>Substitution elasticity (within imports)</td>
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<tr>
<td>Substitution elasticity (within non-resource exports)</td>
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<tr>
<td>Substitution elasticity (across non-resource exports)</td>
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<tr>
<td>Calvo parameter (non-traded goods)</td>
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<tr>
<td>Calvo parameter (imports)</td>
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<tr>
<td>Calvo parameter (non-resource exports)</td>
</tr>
<tr>
<td>Taylor rule parameter (inflation)</td>
</tr>
</tbody>
</table>

24 In concurrent work, we show that in a similar, though slightly simpler model, a weight on inflation of this order of magnitude is similar to that derived when solving for optimal monetary policy under commitment.
Estimation

The remaining parameters of the model are estimated using a Generalized Method of Moments (GMM) procedure. We choose the remaining parameters of the model to match the IRFs between the empirical VAR discussed in Section 2 and the model-implied IRFs. Specifically, we minimise the following measure of distance

$$\hat{\theta} = \arg \min_{\theta} \sum_{j=1}^{5} \sum_{k=1}^{5} \left( g_{jk}^{\text{Model}}(\theta) - g_{jk}^{\text{VAR}} \right) W_{jk} \left( g_{jk}^{\text{Model}}(\theta) - g_{jk}^{\text{VAR}} \right)',$$

where $\theta$ is a parameter vector containing the parameters to be estimated, $j$ relates to the observable being matched (either resource prices, the real exchange rate, inflation, the non-mining GDP to reserves ratio, and the mining capital expenditure to reserves ratio), $k$ denotes the time horizon from to the initial impulse (one being the period in which the resource price shock occurs), $g_{jk}^{\text{Model}}(\theta)$ is the IRF implied by the model evaluated at $\theta$, $g_{jk}^{\text{VAR}}$ is the estimated IRF from the empirical VAR, and $W_{jk}$ is a diagonal matrix that weights the deviations between the model and VAR IRFs by the width of the 95 per cent confidence interval for each IRF point as estimated using the empirical VAR. 25

The results of this estimation procedure are reported in Table 3 and the fit of the best matching model is reported in Figure 2. The importance of domestic ownership for resources firms’ stochastic discount factor is estimated at 0.35 which is not that dissimilar to estimates of domestic ownership within the resources sector (see for example Connolly and Orsmond 2011). The estimated coefficient of relative risk aversion is high at 10, although is in line with the values required to rationalise the equity premium puzzle (see for example Mehra and Prescott and (1985) and Constantinides (1990)). 26

25 An appealing feature of our approach is that when simulating the model for a large number of periods (1000) and estimating a VAR specification that is consistent with that used for the empirical VAR, we are able to recover the simulated model-IRFs to a resource price shock with a small degree of approximation error. The reason that we believe identification is achieved is that we include reserves, a key state variable in our model, in our observed information set. In particular, reserves are used to deflate both non-resource GDP and resource-specific capital expenditure. Using a VAR without deflating real variables by reserves highlighted that a low-order VAR could not recover the simulated model IRFs otherwise.
The elasticity of the foreign risk premium with respect to debt scaled by domestic reserves appears large but this represents the effect of scaling. When considered on the metric of the induced percentage point movement in the foreign risk premium, this parameter appears plausible (see Figure 8). Consistent with the IRFs obtained from the empirical VAR, resource prices are estimated to be a very persistent process with an autoregressive parameter of 0.9.

Interestingly, the data does favour a model where the covariance between firms marginal costs and resource prices is positive, 0.33, and so there does appear to be some input-cost inflation reflecting the correlation between resource and energy prices. Estimates for the parameters regarding investment adjustment costs and exploration costs appear plausible, with the latter suggesting exploration costs increase at a slightly faster rate than discovered reserves. These estimates imply average growth responses of exploration, investment and reserves that are comparable to those observed over the period 1976 to 2011 (Figure 1) given the observed change in resource prices over the same period.

<table>
<thead>
<tr>
<th>Table 3: Parameters Estimated via GMM</th>
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<tbody>
<tr>
<td>Risk aversion coefficient</td>
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<tr>
<td>Domestic ownership parameter</td>
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<tr>
<td>Risk premium (repayment capacity)</td>
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<tr>
<td>Risk premium (valuation dynamics)</td>
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<tr>
<td>Exploration costs dynamics</td>
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<td>Investment cost parameter</td>
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<tr>
<td>Covariance parameter (marginal costs)</td>
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<tr>
<td>AR(1) parameter (prices)</td>
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<tr>
<td>Interest rate smoothing parameter</td>
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</table>

**Results**

Figures 5 to 6 show the resources sector response to a one percent increase in resource prices in general equilibrium and comparing the models with exogenous reserves and endogenous reserves. The first point to note is that the amplification effects associated with the inclusion of endogenous reserves remain present in general equilibrium. The incentive to engage in exploration, which results in

---

26 In the current context, a high CRRA coefficient is required to limit the sensitivity of consumption to a resource price shock. All else constant, a lower value for the CRRA coefficient implies that consumption becomes too volatile.
a permanent increase in resources firms’ future extraction set, does result in
greater labour utilisation, capital intensive investment, and greater production of
resources. It also results in a smaller increase in the shadow value of reserves as
the endogenous exploration response mitigates the marginal value of an extra unit
of reserves.

Nevertheless, it is also clear that in general equilibrium the degree of amplification
attributable to endogenous reserves is smaller when measuring the effects of a
resource price shock on the resources sector. This appears reasonable, and to some
extent should be expected, given that the real exchange rate appreciates offsetting
part of the increase in the value of resource export receipts, that greater demand for labour induces an increase in wages paid in the sector, and that the prices of non-traded inputs also rise in general equilibrium. The latter pricing effects increase the costs of expansion in the resources sector, both in terms of production and exploration, and so the divergence between the IRFs in the endogenous and exogenous models, while still economically significant, are smaller than when compared with the previous partial equilibrium results.

Turning to the rest of the domestic economy, the amplification effects of a resource price shock are also present in the domestic activity responses (Figure 7).
Comparing the responses with endogenous and exogenous reserves respectively, one can see that the decline in consumption (both non-traded and imported) and non-resource exports are larger under endogenous reserves. This is because the expansion in the resources sector absorbs a greater fraction of domestic (non-traded) production, and is consistent with the rise in expected real interest rates required to stabilise the inflationary effects of resource sector expansion. In net terms though, the effect of domestic production is close to zero as the declines in consumption and non-resource export production are fully offset by the expansion of inputs provided to the resource sector.
In contrast, with exogenous reserves, there is less sectoral reallocation. With a smaller expansion in resources sector activity, expected real interest rates rise by less and so consumption and non-resource exports fall by less. In addition to less sectoral reallocation, the net or overall effect on domestic activity is now negative. This is because the income effects tied to higher resource prices are smaller with exogenous reserves, and less than fully offset the decline in domestic demand associated with substitution effects such as higher non-traded goods prices.

For domestic inflation, the domestic interest rate, and the foreign risk premium, we see that the inclusion of endogenous reserves primarily affects the persistence
of responses, but not their relative magnitude (Figure 8). For the real exchange rate, however, the propagation of the shock is largely unaltered with an initial appreciation of the real exchange rate followed by a subsequent depreciation.\textsuperscript{27}

In sum, our findings suggest that the incorporation of endogenous reserves can affect domestic allocations in terms of quantities, with a greater implied expansion in the resources sector being accommodated by a greater contraction in non-resources production. However, the implications for relative prices, including inflation, the real exchange rate, and interest rates are similar regardless of whether an endogenous or exogenous approach to modelling natural resource reserves is used.

5. Conclusion

This paper has studied whether the assumption of an exogenous stock of natural resources is innocuous in the context of a small open economy DSGE model. Our findings suggest that the standard exogenous reserves approximation is reasonable for quantifying the effects of a commodity or resource price shock on key prices of interest including the real exchange rate, inflation and the domestic interest rate.

However, our results also highlight that the standard approach is likely to underestimate the effects of a resource price shock on the resources sector itself, with larger expansions in investment, labour utilisation and production occurring when an endogenous stock of reserves is assumed. Consistent with this, we also find that the sectoral reallocation effects of a resource price shock for the domestic economy are larger under endogenous reserves. This is because the resources sector absorbs more of domestic production and so consumption and non-resource export production both fall by comparatively more. The net effect on domestic economic activity with endogenous reserves is, nevertheless, small.

Our findings also suggest that a relatively simple model with a resources sector, and including only habits and symmetric sectoral price-rigidities, can replicate some of the stylised facts of Australia’s commodity-price boom when viewed through the lens of a simple structural VAR. This suggests that this model has the potential for further empirical applications including simulation of counterfactual

\textsuperscript{27} The responses for the real exchange rate are inverted so that an appreciation in the graph is a movement upwards.
paths for the Australian economy when resource price shocks are the only driver of economic volatility, and to provide additional insight into the welfare effects of a commodity or resource price boom. We plan to investigate these applications, and the implications for macroeconomic policy, in future research.
Appendix A: Analytical Properties of Partial Equilibrium Model

Proposition 2. In the absence of investment adjustment costs, the equilibrium law of motion for reserves

\[ R_{t+1} = R_t + \omega_{t+1} D \left( W_r^t, S_t, P_t^*, P_r^*, R_t, K_t \right) - X \left( W_r^t, S_t, P_t^*, P_r^*, R_t, K_t \right) \]

can be re-written as

\[ \ln R_{t+1} = \ln R_t + \chi^R_t \]

where

\[ \chi^R_t \equiv \ln \left( \left( 1 + \omega_{t+1} d \left( W_t^m, S_t, P_t^*, P_r^*, k_t \right) - x \left( W_t^m, S_t, P_t^*, P_r^*, k_t \right) \right) \right) \]

\[ d_t = \frac{D_t}{R_t} = d \left( W_t^m, S_t, P_t^*, P_r^*, k_t \right) \]

\[ x_t = \frac{X_t}{R_t} = x \left( W_t^m, S_t, P_t^*, P_r^*, k_t \right) \]

\[ k_t = \frac{K_t}{R_t} \]

Proof. In the absence of investment adjustment costs the firms decision problem is given by

\[
V(K_t, R_t) = \max_{I_t(j), D_t(j), X_t(j)} \left\{ \begin{array}{c}
S_t P_r^* X_t - W_r^t \left( X_t^{1+\zeta} K_t^{1-\mu} R_t^{\mu-\zeta} \right) \\
- S_t P_t^* I_t - C \left( D_t, \tilde{R}_t \right) 
\end{array} \right\} + \beta \int V(K_{t+1}, R_{t+1}) d\Phi \left( \xi_{t+1} | \xi_t \right)
\]

subject to:

\[ K_{t+1} = (1 - \delta) K_t + I_t \]

\[ R_{t+1} = R_t + \omega_{t+1} D_t - X_t \]
The associated first-order conditions at an interior solution are

\[ S_t P_t^r = (1 + \zeta) W_r^r x_t^r k_t^{-\mu} + Q_t^R \]
\[ S_t P_t^* = Q_t^K \]
\[ \frac{\partial C}{\partial D_t} = Q_t^R \]
\[ k_{t+1} = (1 - \delta) k_t + i_t \]
\[ r_t = 1 + \omega_{t+1} d_t - x_t \]
\[ Q_t^R = \beta E_t \left( (\zeta - \mu) W_{t+1}^m x_{t+1}^{1+\zeta} k_{t+1}^{-\mu} + Q_{t+1}^R \right) \]
\[ Q_t^K = \beta E_t \left( \mu W_r^r x_t^{1+\zeta} k_t^{-\mu-1} + Q_{t+1}^K (1 - \delta) \right) \]

Note that the solutions to \( x_t, d_t, q_t^R, q_t^K, k_{t+1} \) can be solved from the simplified system

\[ S_t P_t^r = (1 + \zeta) W_r^r x_t^r k_t^{-\mu} + Q_t^R \]
\[ S_t P_t^* = Q_t^K \]
\[ \frac{\partial C}{\partial D_t} = Q_t^R \]
\[ Q_t^R = \beta E_t \left( (\zeta - \mu) W_{t+1}^m x_{t+1}^{1+\zeta} k_{t+1}^{-\mu} + Q_{t+1}^R \right) \]
\[ Q_t^K = \beta E_t \left( \mu W_r^r x_t^{1+\zeta} k_t^{-\mu-1} + Q_{t+1}^K (1 - \delta) \right) \]

yielding the policy functions

\[ x_t = x \left( W_r^r, S_t, P_t^*, P_t^{r*}, k_t \right) \]
\[ d_t = d \left( W_r^r, S_t, P_t^*, P_t^{r*}, k_t \right) \]
\[ Q_t^R = Q_t^R \left( W_r^r, S_t, P_t^*, P_t^{r*}, k_t \right) \]
\[ Q_t^K = Q_t^K \left( W_r^r, S_t, P_t^*, P_t^{r*}, k_t \right) \]
\[ k_{t+1} = k \left( W_r^r, S_t, P_t^*, P_t^{r*}, k_t \right) \]
Noting that these policy functions do not include $R_t$ in their argument, it follows that the law of motion for log reserves will be in equilibrium will be given by

$$\ln R_{t+1} = \ln R_t + \chi_t^R$$

where

$$\chi_t^R \equiv \ln \left( 1 + \omega_{t+1} d \left( W_t^m, S_t, P_t^*, P_t^{*r}, k_t \right) - x \left( W_t^m, S_t, P_t^*, P_t^{*r}, k_t \right) \right)$$

is not a function of reserves and so reserves will have a unit root in equilibrium.

\[\square\]

**Proposition 3.** Assume the firm solves the following dynamic program

$$V(K_t, R_t) = \max_{I_t(j), D_t(j), X_t(j)} \left\{ \left[ S_t P_t^{*r} X_t - W_t^r \left( X_t^{1+\xi} K_t^{1-\mu} R_t^{\mu-\zeta} \right) \right] - S_t P_t^{*} I_t - C(D_t, R_t) \right\}$$

subject to:

$$K_{t+1} = (1 - \delta) K_t + \left( 1 - \Xi \left( \frac{I_t}{I_{t-1}} \right) \right) I_t$$

$$R_{t+1} = R_t + \omega_{t+1} D_t - X_t$$

and where $C(D_t, R_t)$ is homogenous of degree one. In this case the equilibrium law of motion for log-reserves can be also written as

$$\ln R_{t+1} = \ln R_t + \chi_t^R$$

where $\chi_t^R$ is defined in Proposition 2.

**Proof.** This proof follows noting that the profit function is homogenous of degree one when $C(D_t, R_t)$ is homogenous of degree one. Since the constraints are also linearly homogenous, the value function itself is linear homogenous and the associated policy functions are homogenous of degree one (see Stokey and Lucas,
1989, Section 9.3 for a formal treatment). This implies the policy functions can be written as

\[ x_t = x\left(W^r_r, S_t, P_t^*, P_r^r, k_t\right) \]
\[ d_t = d\left(W^r_r, S_t, P_t^*, P_r^r, k_t\right) \]
\[ i_t = i\left(W^r_r, S_t, P_t^*, P_r^r, k_t\right) \]

and so the log-level of reserves in equilibrium will be given by

\[ \ln R_{t+1} = \ln R_t + \ln \left(1 + \omega_{t+1}d_t - x_t\right) \]

where \(d_t\) and \(x_t\) are not functions of \(R_t\) in the solution to the above program. \(\square\)

**Appendix B: Existence of Stationary Representation in General Equilibrium**

**Proposition 4.** A detrended stationary representation of the general equilibrium economy exists if

1. Foreign demand and the stock of domestic natural reserves are cointegrated (share the same long-run relationship)

\[ \ln \frac{Y_t^*}{R_t} / Y_t^* = \vartheta \ln \frac{Y_{t-1}^*}{R_{t-1}} - \Delta \ln R_t + \varepsilon_t \]

2. Resources sector and non-resources sector technology are also identically cointegrated with the stock of domestic natural reserves

\[ \ln \frac{A_t^n}{R_t} / R_t = \vartheta \ln \frac{A_{t-1}^n}{R_{t-1}} - \Delta \ln R_t + \varepsilon_t \]

\[ A_t^n = A_t^r \]
Proof. A detrended stationary representation of the economy in Equations (10) to (18) and (22) to (51) is given by:

\[
\begin{align*}
S_t^* &\approx (1 + \zeta) \frac{W^r_t}{P^e_t} \frac{R_t}{P^e_t} \left( \frac{X_t}{R_t} \right)^\zeta \left( \frac{K_t}{R_t} \right)^{-\mu} + \frac{Q^r_t}{P^e_t} \\
S_t^* &\approx Q^r_t \left( 1 - \frac{I_t}{R_t} \right) \left( \frac{I_t}{R_t} \right) - \zeta' \left( \frac{I_t}{R_t} \right)^2 + \beta E_t \left( \tilde{M}_{t,t+1} \frac{Q^r_{t+1}}{P^e_{t+1}} \right)
\end{align*}
\] (B1)

\[
\frac{\partial C(D_t, R_t)}{D_t} = \frac{Q^r_t}{P^e_t}
\] (B2)

\[
\frac{K_{t+1}}{R_{t+1}} = (1 - \delta) \frac{K_t}{R_t} + \left( 1 - \frac{I_t}{R_t} \right) \frac{I_t}{R_t}
\] (B3)

\[
\frac{R_{t+1}}{R_t} = 1 + \omega_{t+1} \frac{D_t}{R_t} - \lambda \frac{X_t}{R_t}
\] (B4)

\[
\frac{\tilde{M}_{t,t+1}^\zeta}{\tilde{M}_{t,t+1}^\mu} = \beta E_t \left( \tilde{M}_{t,t+1} (\zeta - \mu) \frac{W^r_{t+1} R_{t+1}}{P^e_{t+1} A^r_{t+1}} \left( \frac{X_{t+1}}{R_{t+1}} \right)^{1+\zeta} \left( \frac{K_{t+1}}{R_{t+1}} \right)^{-\mu} \right)
\] (B5)

\[
\frac{Q^r_{t+1}}{P^e_{t+1}} = \beta E_t \left( \tilde{M}_{t,t+1} (\zeta - \mu) \frac{W^r_{t+1} R_{t+1}}{P^e_{t+1} A^r_{t+1}} \left( \frac{X_{t+1}}{R_{t+1}} \right)^{1+\zeta} \left( \frac{K_{t+1}}{R_{t+1}} \right)^{-\mu} \right)
\] (B6)

\[
\frac{Q^r_{t+1}}{P^e_{t+1}} = \beta E_t \left( \tilde{M}_{t,t+1} (\zeta - \mu) \frac{W^r_{t+1} R_{t+1}}{P^e_{t+1} A^r_{t+1}} \left( \frac{X_{t+1}}{R_{t+1}} \right)^{1+\zeta} \left( \frac{K_{t+1}}{R_{t+1}} \right)^{-\mu} \right)
\] (B7)

\[
\frac{\Psi_t^R}{P^e_t R_t} = \frac{S_t^* P^e_t}{P^e_t R_t} \frac{X_t}{R_t} = \frac{W^r_t R_t}{P^e_t A^r_t} \left( \frac{X_t}{R_t} \right)^{1+\zeta} \left( \frac{K_t}{R_t} \right)^{-\mu} - \frac{S_t^* P^e_t}{P^e_t R_t} \frac{I_t}{R_t} - \frac{C(D_t, R_t)}{P^e_t R_t}
\] (B8)

\[
\frac{X_t}{R_t} = \left( \frac{A^r_t}{R_t} H_t^r \right)^\eta \left( \frac{K_t}{R_t} \right)^\gamma
\] (B9)

\[
\frac{D_t}{R_t} = \frac{Y_t^{\mu,n}/R_t}{Q/R_t}
\] (B10)
\[ \Theta_t = \frac{R_t}{C_t} \left( \frac{C_t/R_t - R_t}{V_t/R_{t-1} - R_{t-1}} \right)^{1 - \xi_c} \]  
(B11)

\[ \tilde{\Theta}_t = \beta E_t \left( (1 + i_t) \frac{P_t^c}{P_{t+1}^c} \tilde{\Theta}_{t+1} \frac{R_t}{R_{t+1}} \right) \]  
(B12)

\[ \tilde{\Theta}_t = \beta E_t \left( (1 + i_t^*) \Phi_t \frac{P_t^c}{P_{t+1}^c} \tilde{\Theta}_{t+1} \frac{S_{t+1}}{S_t} \frac{R_t}{R_{t+1}} \right) \]  
(B13)

\[ \frac{W_t^n}{R_t P_t^c} \tilde{\Theta}_t = \varsigma \zeta_h \left( \frac{H_t^0}{H^n} \right)^{1 - \xi_c} \left( \frac{H_t^0}{H^n} + \frac{H_t^r}{H^r} \right)^{\xi_h \gamma_h - 1} \]  
(B14)

\[ \frac{W_t^r}{R_t P_t^c} \tilde{\Theta}_t = \varsigma \zeta_h \left( \frac{H_t^0}{H^n} \right)^{1 - \xi_c} \left( \frac{H_t^0}{H^n} + \frac{H_t^r}{H^r} \right)^{\xi_h \gamma_h - 1} \]  
(B15)

\[ \frac{C_t}{R_t} = \left( 1 - \alpha \right) \frac{C_t^{\delta}}{R_t} \left( \frac{C_t^{\delta}}{R_t} \right)^{\xi_c - 1} + \left( \alpha \right) \frac{C_t^{\nu}}{R_t} \left( \frac{C_t^{\nu}}{R_t} \right)^{\xi_c - 1} \]  
(B16)

\[ \frac{C_t^{\nu}}{R_t} = (1 - \alpha) \left( \frac{P_t^c}{P_t^c} \right)^{-\xi_c} \frac{C_t}{R_t} \]  
(B17)

\[ \ln \chi_t = \rho_r \ln \chi_{t-1} + \Upsilon \epsilon_t^r \]  
(B20)
\[
\begin{align*}
\frac{P_t^n}{P_t^*} &= (1 - \tau_n) \left( \frac{\theta_n}{\theta_n - 1} \right) \frac{V_t^n / (P_t^c) \theta_n}{U_t^n / (P_t^c) \theta_n - 1} \quad \text{(B21)} \\
\frac{V_t^n}{(P_t^c) \theta_n} &= Y_t \Theta_t \left( \frac{P_t^n}{P_t^c} \right)^{\theta_n} W_t^n R_t \frac{R_t P_t^c A_t^n \chi_t}{R_t P_t^c A_t^n \chi_t} + \beta \phi_n E_t \left( \frac{V_{t+1}^n}{(P_t^c) \theta_n} \left( \frac{P_{t+1}^c}{P_t^c} \right)^{\theta_n} \right) \quad \text{(B22)} \\
\frac{U_t^n}{(P_t^c) \theta_n - 1} &= Y_t \Theta_t \left( \frac{P_t^n}{P_t^c} \right)^{\theta_n} + \beta \phi_n E_t \left( \frac{U_{t+1}^n}{(P_t^c) \theta_n - 1} \left( \frac{P_{t+1}^c}{P_t^c} \right)^{\theta_n - 1} \right) \quad \text{(B23)} \\
\frac{P_t^n}{P_t^*} &= \left( 1 - \phi_n \right) \left( \frac{P_t^n}{P_t^*} \right)^{1 - \theta_n} + \phi_n \left( \frac{P_t^n}{P_t^c} \right)^{1 - \theta_n} \frac{1}{1 - \theta_n} \quad \text{(B24)} \\
\frac{\Psi_t^n}{R_t P_t^c} &= Y_{t \cdot n} \Theta_t \left( \frac{P_t^n}{P_t^c} \right)^{\theta_n} + \phi_n \left( \frac{P_t^n}{P_t^c} \right)^{1 - \theta_n} \frac{1}{1 - \theta_n} \quad \text{(B25)} \\
\frac{V_t^o}{(P_t^c) \theta_o} &= C_t \Theta_t \left( \frac{P_t^o}{P_t^c} \right)^{\theta_o} S_t P_t^* + \beta \phi_o E_t \left( \frac{V_{t+1}^o}{(P_t^c) \theta_o} \left( \frac{P_{t+1}^c}{P_t^c} \right)^{\theta_o} \right) \quad \text{(B27)} \\
\frac{U_t^o}{(P_t^c) \theta_o - 1} &= C_t \Theta_t \left( \frac{P_t^o}{P_t^c} \right)^{\theta_o} + \beta \phi_o E_t \left( \frac{U_{t+1}^o}{(P_t^c) \theta_o - 1} \left( \frac{P_{t+1}^c}{P_t^c} \right)^{\theta_o - 1} \right) \quad \text{(B28)} \\
\frac{P_t^o}{P_t^*} &= \left( 1 - \phi_o \right) \left( \frac{P_t^o}{P_t^*} \right)^{1 - \theta_o} + \phi_o \left( \frac{P_t^o}{P_t^c} \right)^{1 - \theta_o} \frac{1}{1 - \theta_o} \quad \text{(B29)} \\
\frac{\tilde{P}_t^o}{P_t^*} &= \left( 1 - \phi_o \right) \left( \frac{P_t^o}{P_t^*} \right)^{- \theta_o} + \phi_o \left( \frac{\tilde{P}_t^o}{P_t^c} \right)^{- \theta_o} \frac{1}{1 - \theta_o} \quad \text{(B30)} \\
\frac{\Psi_t^o}{R_t P_t^c} &= Y_{t \cdot o} \Theta_t \left( \frac{P_t^o}{P_t^c} \right)^{\theta_o} + \phi_o \left( \frac{\tilde{P}_t^o}{P_t^c} \right)^{- \theta_o} \frac{1}{1 - \theta_o} \quad \text{(B31)} 
\end{align*}
\]
\[ P_t^* = (1 - \tau_x) \left( \frac{\theta_x}{\theta_x - 1} \right) V_t^x \]

\[ V_t^x = \frac{C_t^*}{R_t} \left( P_t^* \right)^{\theta_x} \frac{P_t^c}{P_t^x} + \beta \phi_x E_t (V_{t+1}^x) \]

\[ U_t^x = \frac{C_t^*}{R_t} \left( P_t^* \right)^{\theta_x} S_t \frac{P_t^c}{P_t^x} + \beta \phi_x E_t (U_{t+1}^x) \]

\[ P_t^* \equiv \left( (1 - \phi_x) \left( P_t^* \right)^{1-\theta_x} + \phi_x \left( P_{t-1}^x \right)^{1-\theta_x} \right)^{-\frac{1}{1-\theta_x}} \]

\[ \tilde{P}_t^x = \left( (1 - \phi_x) \left( P_t^* \right)^{-\theta_x} + \phi_x \left( \tilde{P}_{t-1}^x \right)^{-\theta_x} \right)^{-\frac{1}{\theta_x}} \]

\[ \Psi_t^x \equiv S_t \frac{P_t^x}{P_t^c} C_t^* - (1 - \tau_x) \frac{P_t^c}{P_t^x} \left( \frac{\tilde{P}_t^x}{P_t^x} \right)^{-\theta_x} \frac{C_t^*}{R_t} \]

\[ \ln(1 + i_t) = (1 - \rho) \ln(1 + \bar{i}) + (1 - \rho) \rho \ln(\frac{P_{t+1}^c}{P_t^c}) + \rho_1 \ln(1 + i_{t-1}) \]

\[ \frac{A^n_t H^n_t}{R_t \chi_t} = \left( \frac{\tilde{P}_t^c / P_t^c}{P_t^c / P_t^x} \right)^{-\theta_n} \frac{Y_{d,n}^t}{R_t} \]

\[ \frac{Y_{d,n}^t}{R_t} \equiv C_t^n + \left( \frac{\tilde{P}_t^c}{P_t^x} \right)^{-\theta_x} \frac{C_t^*}{R_t} + \frac{Y_{r,n}^t}{R_t} - D \frac{Q^r}{R \phi_{mc}} \]

\[ \tilde{P}_t^c \equiv \left( (1 - \phi_n) \left( \tilde{P}_t^c / P_t^c \right)^{-\theta_n} + \phi_n \left( \tilde{P}_{t-1}^c / P_{t-1}^c \right)^{-\theta_n} \right)^{-\frac{1}{\theta_n}} \]

and where

\[ \tilde{M}_{t,t+1} = \left( \frac{\Theta_{t+1}^r}{\Theta_t^r} + 1 - \nu \right) \]

Note that cointegration between foreign demand between for non-resource exports and the stock of reserves implies that the ratio \( Y_t^*/R_t \) will be stationary, and similarly, that the (identical) cointegration between domestic technology \( A_t^n = A_t^r \) and the stock of reserves implies that the ratios \( A_t^n/R_t \) and \( A_t^r/R_t \) are stationary. The system of Equations (B1) to (B41) describe a stationary detrended economy given by the quantities
Appendix C: Further discussion on the Resources Problem

One important distinction between the above dynamic program and the approach described in Bohn and Deacon (2000) is the absence of a finite upper bound on the cumulative level of resources that can be discovered. This abstraction is important for our analysis since it implies that the policy functions that solve the above problem are time invariant and admit a stationary (detrended) non-stochastic steady state. Intuitively, our approach implies that firm decisions concerning investment, exploration and production are not substantively affected by the knowledge of a known finite level of reserves still to be discovered. This appears to be a reasonable assumption, at least in the Australian context. It is a different problem, however, from that in which a natural resources firm simply chooses its allocations of labour, capital and production to optimally extract from a pre-defined finite resource stock over time (whether exploration is required or not).

Although we abstract from a known finite bound on the remaining stock of undiscovered reserves, we do not entirely abstract from the concept of resource scarcity. As an alternative, we assume that the costs associated with exploration activity are increasing in the quantity of previously accumulated aggregate reserves. Specifically, we assume that the cost function is increasing in both exploration and aggregate reserves, \( \frac{\partial C}{\partial R_t} > 0 \), \( \frac{\partial C}{\partial D_t} > 0 \), that the derivative of the marginal cost of exploration is increasing in the level of exploration, \( \frac{\partial^2 C}{\partial D_t^2} > 0 \), and that this same derivative is sufficiently large that it outweighs any possible reduction in the marginal costs of exploration that might be tied to greater existing reserves permitting more delineation of (or new finds linked to) existing deposits \( \frac{\partial^2 C}{\partial D_t \partial R_t} > 0 \). Finally, we assume that exploration costs tend to infinity as the stock of accumulated reserves becomes large \( \lim_{R_t \to \infty} C(D_t, \tilde{R}_t) = \infty \).

Together, these assumptions are consistent with many of the approaches adopted in the natural resources literature including for example Heal (1993), Pindyck (1978),...
Sweeney (1993), and Reiss (1990). Although this literature covers a much wider range of cases, than we have time to discuss here, and the quantitative implications may well be different depending on the precise structure used, our own view is that the above restrictions capture the essence of a resources firm problem. An appealing feature of approach, and indeed our main motivation, is that the above problem specified in this way is directly integrable in small-open-economy DSGE model. This is of primary importance given our aim is to understand how a more richly specified resources sector could affect the propagation of resource price shocks.
References


