A Multi-Sector Model of Growth with Innovation Clustering

Steven Bond-Smith

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Abstract

We develop a multi-sector model with endogenous growth through quality improving innovations where technologically related firms prefer to co-locate in clusters due to technical externalities in R&D. Each firm’s technology sector and the location of other firms plays a role in each firm’s ability to improve its own technology. As a result, firms prefer to co-locate in technologically grouped clusters. Technology growth is highest in clustered and agglomerated locations. Production of each variety is contestable through vertical innovation and firms choose a sector, variety and location to maximize profit by maximising the productivity of R&D. The direction and magnitude of the clustering effect is specific to each sector depending upon the spatial distribution and knowledge intensity of all firms and sectors. Firms choose an optimal location for R&D by balancing the clustering effect with the manufacturing agglomeration effect. Sectors can cluster in peripheral locations if the sector has sufficiently greater knowledge spillovers from their own sector. However technology changes mean peripheral clusters are typically only sustainable in the medium term. Extending the model to include additional location factors of production means a firm could choose a less than optimal location for R&D but interior sector equilibria are only short term. The model suggests that sustaining peripheral clusters requires a key role from government in monitoring performance and potentially assisting the R&D sector.

1 Introduction

Endogenous growth theory recognizes the role of knowledge spillovers to research and development (R&D) decisions, encouraging agglomeration and implying the important role of clustering to innovation and growth. Mechanisms which lead to industrial clustering (and concentration of firms and people) have been considered since Marshall (1890) including knowledge spillovers, transport costs and factor prices. While the literature has long recognized factors that encourage clustering, modern endogenous growth theories only imply its importance and fail to incorporate the proximity characteristics of knowledge and innovation which encourage industrial clustering. Knowledge spillovers between firms vary
with the concentration of people, production and related technologies. Clustering of technologically related firms has been established as a mechanism to facilitate technology spillovers between technologically similar firms. Endogenous growth theory should consider how clustering forces interact with innovation and growth. Agglomeration and knowledge spillovers re-enforce each other with circular causality in regional models of growth yet these typically ignore the formation or role of clustering.

Models of growth and agglomeration inevitably describe the formation and continued expansion of large cities and urban economics has long eluded to the concept of specialized cities. Clusters maintain technological advantages over other locations and further encourage the co-location of technologically related firms. What policy choices (if any) are available for declining (relatively) or lagging regions to keep up with modern globally connected agglomerated or specialised cities? We consider a multi-sector model of growth where the cost of innovation is dependent on the location and technology of all firms and the selected level of technological improvement. It is possible for industries to cluster in a smaller region due to hysteresis even though innovation is more expensive than if all industries were agglomerated in the same location.

A range of models combine spatial factors with the new growth theory literature. These models attempt to explain variations in economic growth between regions and nations through both pecuniary and knowledge spillover externalities. Several models combine increasing product variety (Romer, 1990; Grossman and Helpman, 1991a) with the new economic geography (Baldwin et al., 2001; Baldwin and Forslid, 2000; Martin and Ottaviano, 1999; Martin, 1999; Fujita and Thisse, 2003) predominantly due to the fundamental common use of Dixit-Stiglitz (1977) competition. The different models vary assumptions on the mobility of capital, labor and intermediate versus consumer demand to influence the forward and backward linkages. However, at low transport costs, catastrophic agglomeration is inevitable in these models with mobile capital and/or labor. While NEG and growth models typically feature agglomeration as a characteristic of equilibrium, clustering is a rare feature. In addition, we consider that technology levels should be recognised as a key factor in location decisions related to knowledge spillovers. Since growth in these models comes from an increasing variety of products, this range of models is unfit to consider the role of technology levels in R&D spillovers.

Some trade models include comparative advantage but in these models agglomeration is caused by vertical linkages with intermediate goods used in production and the pecuniary externality of transport costs. For example, Venables (1999) creates a model of international specialization by extending Krugman and Venables (1996) where two industries both use and produce intermediate goods. In the model clustering is due to pecuniary externalities in manufacturers’ consumption of intermediate goods rather than technical externalities in knowledge transfer as found in the innovation economics literature. Yamamoto (2003) has partial spatial knowledge transfer externalities in the R&D sector and pecuniary externalities in R&D consumption of intermediate goods but with only one manufacturing sector the model must prevent labor mobility to
avoid catastrophic agglomeration and cannot describe industrial clustering.

Products are today increasingly manufactured using a multitude of components produced in many countries. With low transport costs and many suppliers and locations, pecuniary externalities from transport costs are becoming less relevant to firm location choices and trade. Despite the apparent ability to operate anywhere, the world is not flat; it is increasingly uneven. The innovation input to final goods production is dependent on knowledge inputs and not all knowledge is directly transferable between locations. This increases the premium associated with face-to-face contact (Gaspar and Glaeser, 1998; Storper and Venables, 2004; McCann, 2008; Rodríguez-Pose and Crescenzi, 2008) because of technical externalities in R&D on the inputs side of the production process in knowledge intensive industries. Cities are becoming more important to production due to the nature of knowledge and technology. Therefore it is necessary to consider a trade and growth model where knowledge transfer and innovation are at the centre of explaining specialized innovation clusters and agglomerations. In this type of model knowledge transfer externalities are the key cause of clustering and agglomeration.

Evolutionary economics describes economic development and region specialization as a branching process (Frenken and Boschma, 2007). This conceptual framework goes in the direction we are considering here within a theoretical economic model. The firm’s ability to develop new innovations is related to both its technological and spatial proximity (Boschma, 2005). This type of mechanism gives technologically related firms an incentive to locate in clusters but has not yet been incorporated into endogenous growth models.

Duranton (2007) describes a model of specialised locations by applying the Grossman and Helpman (1991b) quality ladders model to an urban framework to study industry churn. The model uses strictly local knowledge spillovers from the quality leader within each industry only and industry churn is generated by a stochastic process in R&D ignoring any interdependence between research firms. As a result, cities are specialised clusters of industries, but stochastic innovations in other industries lead to the shifting of industries between cities. Alternatively Brezis and Krugman (1997) use an urban model with learning by doing to explain technology lock-in within clusters such that the stochastic emergence of new technologies in other locations cannot be adopted by locked-in incumbents. This leads to the emergence of new clusters and the constant stochastic churning of industries from agglomerations to new clustered locations. These models are inappropriate for studying any relationship between clustering and growth because there are no endogenous forces for industry location. Industry location is instead determined stochastically and maintained by hysteresis. But research firms across all industries use some shared knowledge and technology to develop innovations. Rather than being stochastic, changes in the location of industries are dependent upon the proximity of related technologies. This has implications for economic growth and development policy and is essential to endogenous growth.

What is lacking in these models is any relationship between technologically related sectors or technology levels with R&D. They claim the shifting of in-
dustry is by the accidental emergence of new technologies. But we are able to
develop a model where the shifting of clusters is not accidental. New technologies
will typically occur in related sectors. Futhermore, neither of these models is
designed to consider growth policy in the context of clustering. Here we develop
a model where clustering is a characteristic of the steady state and the shift-
ing of sectors to the agglomeration is endogenous due to technology advances
in agglomerated regions, dependent upon the characteristics of each sector and
the spatial equilibrium. Endogenous growth theory implies the importance of
technology spillovers but existing models fail to consider the role of proximity
in facilitating technology transfer and its effects on economic growth policy.

With modern communication technologies, knowledge spillovers are also not
restricted to the local city area. The creative destruction/quality ladders ap-
proach Grossman and Helpman (1991b); Aghion and Howitt (1992); Young
(1998) offers an opportunity to develop a model where firms make use of knowl-
edge from many sources and locations across both geographic and technological
space. In these models growth comes from increasing product quality instead of
product variety where the cost of improving quality depends on the availability
of knowledge. We develop a model with interdependencies between research
firms due to their common knowledge inputs, even between different locations.
As a result there is the possibility of technologically advanced clustered locations
within an endogenous growth model.

Our starting point is Young (1998), a deterministic quality ladders model
without scale effects where intertemporal knowledge spillovers determine the
cost of innovation. Bond-Smith (2012) extends this model to recognize the
spatial characteristics of knowledge spillovers and innovation, but with only one
manufacturing sector, full agglomeration is inevitable at low transport costs. We
now include multiple manufacturing sectors and develop the knowledge spillovers
mechanism further by following a proximity (Boschma, 2005) approach. This
facilitates the innovation clustering of firms with related technology in an en-
dogenous growth model. Importantly the model allows the possibility for firms
within each industry to cluster in a location other than a large agglomeration.
These industries can remain in the “periphery” in the steady state even with
factor mobility, a feature not possible with previous models of growth and loca-
tion. For simplicity, we remove transport costs to focus solely on the clustering
and agglomeration effects of technical externalities in R&D. The importance of
transport costs to firm location is also already well documented in the trade
and New Economic Geography literature and is not the purpose of this paper.
While there are many factors which firm’s balance when deciding on a produc-
tion location we focus on the trade-offs for firm location decisions based on the
location of other firms and spatial technical externalities in R&D. Even with
zero transport costs, agglomeration effects may be overcome by clustering effects
in peripheral clusters as firms prefer to co-locate alongside related technology
firms. In extending the model we re-introduce trade costs to model the trade
off between multiple spatial factors in firm location decisions.

Section 2 describes the structure of the model, the forces for clustering and
the allocation of industry in the steady state. Section 3 describes the outcomes
for growth in various steady states using simulations. Section 4 reintroduces trade costs such that firms and workers balance forces for clustering with traditional home market effects. Section 5 concludes with policy implications for lagging locations.

2 The Model

We present an economy where firms invest in R&D in order to produce quality improvements and manufacture products for consumers in multiple locations. Technological knowledge related to all existing quality levels is an input to R&D but this knowledge does not transfer perfectly between locations or varieties. The usability of knowledge for innovation is associated with the spatial and technological proximity of related varieties. Firms which choose to locate alongside other manufacturers with technologically related varieties have a lower cost of innovation. Firms choose their location by considering knowledge spillovers, the location of other firms and their technological relatedness with this last factor being the mechanism for industrial clustering.

Consider an economy with two production sectors, traditional and manufactured goods, that each has its own factor of production: unskilled and skilled labor respectively. In the manufacturing sector each firm uses skilled labor in R&D to develop a quality improvement for a single variety from one of many industrial subsectors. If a firm develops a substantive quality improvement they are awarded with a patent giving the firm the monopoly right to produce their variety at that quality level. The patent expires when a successful innovator manages to develop the following quality step. If a firm is able to have the greatest quality improvement for variety \(i\), they are able to “take the market”, producing that variety for the following period. “Taking the market” is where a firm develops the best quality improvement of a variety and supplies the entire “niche” market. If they invest too little in R&D, there is an opportunity for an alternative producer to research the quality improvement for that variety and take the market from the incumbent manufacturer.

Each period a quality improvement destroys the value of the existing quality level because a firm cannot maintain its niche market without R&D. This is the process of “creative destruction” as described by Aghion and Howitt’s (1992) endogenous growth model. Aghion and Howitt (1992) use a contestable monopoly market for an intermediate product. We also use the contestable market idea but with multiple industrial sectors, varieties and monopolistic competition where each variety competes with other varieties in their own sector through Dixit-Stiglitz preferences. Dixit-Stiglitz preferences however are not integral to the model, as growth does not come from increasing product variety, but are a simple way of modelling monopolistic competition - other forms of monopolistic competition would yield a similar outcome.

Existing knowledge is an input to innovation. Greater knowledge inputs reduce the cost of developing a quality improvement by innovation. Knowledge from the same variety is directly transferable to developing quality improve-
ments in the following period. In addition, knowledge associated with the level of quality for other varieties is partially additive to knowledge inputs. If there is a quality improvement in one variety, it provides some knowledge input to other varieties when they develop quality improvements in future periods. Furthermore, knowledge from other firms is not equally weighted. That is, knowledge from manufacturers within a firm’s related technology sector is weighted higher than knowledge from other sectors. Lastly, knowledge is only partially transferable between locations; if a variety is produced in one location, the knowledge associated with its quality level does not fully transfer to the other location as an input to developing quality improvements.

The result is firms choose to locate close to other manufacturers that share the same knowledge inputs. Agglomeration economies in manufacturing are strengthened due to technical knowledge externalities in research as in Baldwin and Forslid (2000). Knowledge spillovers provide additional growth. As in the other growth models, we find growth is highest when there is full agglomeration in one location, as knowledge spillovers are greater with manufacturing concentration. Agglomerated locations are more reliant on local inter-varietal knowledge spillovers for growth while less agglomerated locations rely on trade, regional knowledge spillovers and local knowledge spillovers within clusters. Traditional goods production is shared between locations and included in the model to ensure trade, even with full manufacturing agglomeration.

The model here includes migration of skilled workers (or footloose skilled labor) because we are interested in understanding innovation and knowledge spillovers, in economies where migration may also be a key mechanism for knowledge transfer (Faggian and McCann, 2009). Allowing worker migration gives a better insight into the impact on growth of closer economic integration (beyond trade) between countries and regions. Migration of skilled workers between locations, in response to differences in real wages, equalizes real wages in the long run and trade allows consumers in both locations to benefit from innovative, increasingly higher quality products. With zero transport costs real wages are equal to nominal wages simplifying the analysis. However wage differences may emerge temporarily as certain locations become more advantageous for innovation in specific industries creating greater demand for skilled labour, but nominal (and real) wages equalise in the steady state. As such we can gain a better understanding of the influence on economic growth from migration, competition, location, trade, knowledge and innovation which are all affected by closer economic integration between regions and nations. While footloose labor usually makes catastrophic agglomeration inevitable if there are forces for agglomeration, this is not the case here. Industrial clusters are able to exist in the periphery due to hysteresis and we can consider growth outcomes and policy with an unequal distribution of economic activity, despite factor mobility.

The model here is highly stylised but it shows what is happening in high technology, high knowledge sectors where skilled labor is internationally mobile, firm location is not permanent and transport costs are diminishing in relevance to firm location decisions. We can then consider how this impacts on growth in different types of locations.
2.1 Model Specification

There are two types of labor: unskilled and skilled labor. As in Grossman and Helpman (1991b) and Young (1998) we have a manufacturing sector and a competitive R&D sector that both employ skilled labor. Unskilled labor is employed in the traditional goods sector. Unlike similar models, we extend the model to multiple sectors in manufacturing. We use the multi-sector Cobb-Douglas approach of Venables (1999) with Dixit-Stiglitz preferences in each sector. However in the model here comparative advantage is not caused by vertical linkages with intermediate goods but by technical externalities in R&D.

There are two locations where firms and workers (consumers) can choose to locate. Consumers have a preference for goods in all manufacturing sectors, for traditional goods and for variety in each manufacturing sector. In the basic model we have zero transport costs so consumers demand home and foreign varieties equally.

In each period a manufacturing firm employs skilled workers to produce the firm’s variety with a given quality level for which the firm has a patent. Firms also employ skilled workers in the R&D sector to use existing knowledge of quality levels for all varieties and develop a quality improvement for a variety that the firm will produce in the following period. As in Young (1998) production of each variety is contestable through these quality improvements produced by a competitive R&D sector. For a large enough quality improvement the firm is granted a patent to produce that variety.

As in all endogenous growth models, knowledge spillovers have a vital role. New innovations are based on existing knowledge. We take the approach that knowledge spillovers transfer imperfectly between firms based on technological proximity. We maintain the the approach of Baldwin et al. (2003) that knowledge transfers imperfectly between firms that are spatially separated. We weight the knowledge input from other varieties according to a related variety (Boschma and Frenken, 2009) approach where the technological relatedness of products describes how useful the knowledge is to innovation in a firm’s own variety. We put this into effect by assuming that the knowledge input to innovation from within the firm’s own sector is weighted higher than knowledge from other sectors. For simplicity we assume varieties in the same sector are weighted equally and varieties in other sectors are also weighted equally. A modified version of the model could include individual weights for every pair of related varieties but for simplicity we assume only two possible levels of relatedness. This technical externality in R&D induces firms to cluster in locations alongside their own sector. We describe this as the clustering effect. Firms must also consider this effect alongside incentives to locate in a larger agglomeration where there are more sources of knowledge spillovers. We term the incentive to locate with the larger share of manufacturing, the agglomeration effect.
2.1.1 Preferences

The representative consumer has a taste for traditional goods and a range of manufactured varieties. Consumers have a preference for higher quality manufactured varieties. The two regions are referenced by home and foreign. Where it is necessary to specify foreign variables these are denoted by a tilde. We set the model out for the home region only and analogous equations apply to the foreign region. With $\rho > 0$ as the discount rate, the representative consumer has inter-temporal preferences given by:

$$U = \sum_{t=0}^{\infty} \alpha^t \ln Q_t, \alpha = 1/(1 + \rho)$$ (1)

where $Q_t$ is consumption of traditional (subscript T) and manufactured goods from M sectors, in period $t$ with Cobb-Douglas preferences:

$$Q_t = C_{1-\mu, T}^{1-\mu} \prod_{i=1}^{M} C_{i, t}^{\mu_i}, 0 < \mu < 1$$ (2)

While we are most interested in the knowledge sectors characterized by manufactured goods with increasing quality levels, the traditional goods sector is included in order to describe what is going on in the rest of the economy. It accounts for the less mobile sectors where location is not so important to innovation. In the real world workers in the traditional goods sector are not necessarily unskilled or immobile but these are common properties of the traditional sector in trade models. When studying growth here we also include traditional goods and immobile workers for several reasons. A portion of the workforce in the real world simply does not want to move so it is important that in these models some residual demand remains in all locations, even if there is full agglomeration, so that regions continue to trade. Some factors of production in the real world such as land, are immobile; while we don’t include land as a factor of production the immobility of unskilled workers is a proxy for immobile production factors like land.

That is not to say the types of clustering effects we describe in high-tech sectors are not also occurring in lower technology or traditional sectors, we simply make these assumptions in order to model the general equilibrium. The important feature of the traditional goods sector in the model is that the factor of production is immobile and some portion of all consumption is immobile; “unskilled labor” (or “peasants”) is the commonly used term for this factor in trade models. We only address the traditional goods sector briefly and otherwise the the remainder of the analysis focuses on the knowledge sectors of innovation and manufacturing.

For simplicity, the time subscript $t$ will be suppressed hereafter where the time dimension is clear. Each of the $M$ industrial sectors is modeled as monoplistically competitive via a Dixit-Stiglitz (1977) preferences. The additional factor $A_{i,j}$ represents a further factor of differentiation within each sector: the quality of variety $j$, in industry $i$: 
\[ C_i = \left[ \sum_{j \in i, \tilde{i}} (A_{i,j} c_{i,j})^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}, \quad \sigma > 1 \] 

(3)

c is the quantity consumed, subscript \( i \) indicates each sector, and \( j \) indicates the individual varieties of products within sector \( i \). Varieties can be produced by domestic manufacturers or imported from foreign manufacturers where \( n_i \) and \( \tilde{n}_i \) are the number of home and foreign manufacturers in sector \( i \). Where it is necessary to indicate the difference between the locations of consumers and manufacturers, a tilde on the variable will refer to the location of a foreign consumer and a tilde on the subscript refers to the location of a foreign Manufacturer. Thus a variety produced and consumed in the foreign region is \( \tilde{c}_{i,j} \) while a variety produced in the foreign region but consumed in home is \( c_{i,j} \). For simplicity it will be referred to as variety \( i, \tilde{k} \). With symmetry for all varieties in each sector \( i \) we can drop the variety subscripts \( j \) and \( \tilde{k} \) and simplify the sum (and the tilde is moved to the \( i \) to denote the location of production). Note while \( j \) and \( \tilde{k} \) are different varieties, \( i \) and \( \tilde{i} \) are the same sector group and refer to individual varieties produced in different locations. However in the steady state, each sector will be in only one location, i.e. it will be \( i \) or \( \tilde{i} \) such that either \( n_i \) or \( n_{\tilde{i}} \) is equal to zero. As such equation 3 can be simplified to:

\[ C_i = \left[ n_i (A_i c_i)^{\frac{\sigma - 1}{\sigma}} + n_{\tilde{i}} (A_{\tilde{i}} c_{\tilde{i}})^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \] 

(4)

Where possible variety \( i, \tilde{k} \) is now referred to as a single variety in sector \( \tilde{i} \). We could also think of there being a continuum of manufacturing varieties, sectors and continuous time (replacing Sigmas \( \sum \) with integral signs \( \int \) and \( \prod_{i=1}^{M} \) with the product integral \( \prod_{0}^{M} \int di \)) but it is easiest to set the model up with a discrete, finite number of varieties, sectors and discrete time periods.

Intertemporal utility optimization implies the transversality condition and Euler equation:

\[ \frac{E_t + \tilde{E}_t}{E_{t-1} + \tilde{E}_{t-1}} = \frac{1 + r}{1 + \rho} \] 

(5)

where \( E_t + \tilde{E}_t \) is world consumer expenditure in period \( t \), \( \rho \) is the rate of time preference and \( r \) is the rate of return on savings between periods \( t - 1 \) and \( t \). We normalise world expenditure \( E + \tilde{E} = 1, \forall t \).

Each variety competes only with its own sector via CES preferences\(^1\). For simplicity we assume zero transport costs such that the price paid and the amount of each variety that arrives at its destination is the price received by

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\(^1\)We use CES competition for simplicity. Ottaviano et al. (2002) point out the limitations of CES in a trade model but the focus here is the inputs to innovation and the outcomes for growth rather than retail price competition. Other forms of competition would yield more realistic picture of price competition but would have similar implications for innovation and firm location and would involve a more complicated derivation.
the manufacturer and the amount produced. This allows us to focus exclusively on the location and growth effects of technical externalities in R&D. In an extension of the basic model we later include trade costs to demonstrate how firms balance many factors in making location and investment decisions.

2.1.2 Labour

The world is endowed with one unit of labour, both skilled and unskilled. We follow Krugman (1991) and set the worldwide stock of skilled workers to $\mu$ and the stock of unskilled workers to $(1 - \mu)$, shared equally between regions\(^2\).

With $L_T$ and $\tilde{L}_T$ the supply of unskilled workers in the home and foreign regions respectively is:

$$L_T = \frac{1 - \mu}{2}, \quad \tilde{L}_T = \frac{1 - \mu}{2}$$

so that total production of traditional goods is also shared equally between regions. Unskilled workers cannot migrate between locations.

$L_K$ describes the number of skilled workers in the home region (subscript $K$ describes the “knowledge sectors” of manufacturing and R&D) such that:

$$L_K + \tilde{L}_K = \mu$$  \hspace{1cm} (7)

Manufacturing and R&D are subject to footloose labor where the migration of skilled workers responds to “wage pressure”. If there are wage differences between regions at the beginning of a period there will be migration of skilled workers. The change in skilled workers per period in the home region is given by the migration equation:

$$\dot{L}_K = (w_K - \tilde{w}_k)s(1 - s)$$

where $\dot{L}_K$ is the migration of skilled workers at the start of each period shifting from the foreign region to the home region in response to differences in real wages (or from the home region to the foreign region if $\dot{L}_K$ is negative), $s$ is the share of skilled workers in the home region and $w_K$ is the nominal wage of skilled workers in the home region. With zero transport costs there is no need to define real wages separately from nominal wages for the purpose of migration because there are no price/wage differences between regions. Workers migrate to the region that provides the highest wage rate which is able to purchase those workers the highest level of utility.

\(^2\)The choice of units ($\mu$ skilled workers and $1 - \mu$ unskilled workers) follows Krugman (1991) to ensure that prices and wages in the traditional goods sector are the numeraire, and that the wage rate of unskilled workers equals that of skilled workers. If the number of skilled workers were specified differently, the wages of skilled workers are a constant multiple of the wage rate of unskilled workers. Similarly we could specify a portion of skilled labor as highly skilled to work in R&D only and the wages of highly skilled workers would be a constant multiple of the skilled and unskilled workers’ wages. We maintain simplicity (heterogenous workers are not the focus of our model) by avoiding these additional multiples. A scaling factor could also be used to calibrate the model to any arbitrary growth or wage rate.
2.1.3 Technology

Endogenous growth is of the form in Young (1998) with a multi-sector version of the knowledge spillover mechanism in Bond-Smith (2012). The mechanism can be easily extended to multiple varieties and sectors and the innovation cost function allows endogenous growth by vertical innovations without scale effects. We purposely use a model of growth without scale effects so the clustering effect is solely due to the knowledge spillover externality in R&D and firms do not cluster due to the scale effects present in most endogenous growth models.

In each variety there is a quality leading firm that developed a quality improvement for that variety in the previous period. In each period the quality leader produces for its niche monopoly of variety $i,j$ and conducts R&D to ensure a quality improvement large enough to maintain its niche monopoly position for the following period. A firm must conduct R&D to ensure a quality improvement great enough to remain the producer of variety $j$ (or great enough to take the market for variety $j$ from the incumbent firm)\(^3\). Production of an individual variety involves a fixed (labor) investment in innovation (in the previous period) and a constant marginal cost. The skilled labor requirement as an input to R&D in the previous period, $t-1$, and the fixed cost of innovation to achieve the targeted quality level $A_{i,j,t}$ in period $t$ is:

$$F_{i,j,t-1}(A_{i,j,t}, \bar{A}_{i,j,t-1}) = \begin{cases} 
\gamma e^{\eta A_{i,j,t}/\bar{A}_{i,j,t-1}} & \text{if } A_{i,j,t} \geq \bar{A}_{i,j,t-1} \\
\gamma e^{\eta} & \text{otherwise}
\end{cases}$$

(9)

where $\gamma$ and $\eta$ are constants used for calibration and $\bar{A}_{i,j,t-1}$ is an index of technological opportunity for variety $i,j$, representing the intertemporal spillover of knowledge available to variety $i,j$ researchers from previous generations of all varieties. The fixed cost can be thought of as two components: a standard fixed cost of $\gamma e^{\eta}$ irrespective of quality improvement and a research cost of $\gamma e^{\eta}A_{i,j,t}/\bar{A}_{i,j,t-1} - \gamma e^{\eta}$.

There is free entry for previously produced quality levels below the current quality leader. All firms produce one unit of output per $\beta$ units of labor. For any firm producing a variety at the non-leading quality level, free entry and unit labor costs lead prices to equal $\beta$ divided by the wage rate. With perfect elasticity between multiple providers of a single variety, consumers will purchase each variety that has the highest quality to price ratio. The quality leader is able charge a higher limit price and supply the entire market because consumers prefer the variety with the highest quality to price ratio. We assume parameters such that quality improvements are large enough that this limit price is always

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\(^3\)Duranton (2007) uses a stochastic quality ladders model where an incumbent never innovates in their own variety because of creative destruction; the firm doesn’t want to destroy the value of their existing patent by developing a quality improvement. In the deterministic model here the firm knows a rival will develop a quality improvement at the end of each period so an incumbent can be the innovator in order to continue as the quality leader in the following period.

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greater than the profit maximising monopolist’s price. As a result technology
leading firms will have the highest quality to price ratio at the profit maximising
price and there are no producers at the non-leading quality level; only the highest
quality version of each variety is produced in any period and any former lower
quality versions of that variety are no longer produced. This is the contestable
market for each variety because only the quality leader in each variety can
produce each period. Therefore all productive firms were innovators in the
previous period, were granted a patent to produce in the current period and
each variety is produced by a different manufacturer.

The model described differs from existing growth models in that the addition
of multiple locations and multiple sectors allows us to more realistically model
the spillover of knowledge between firms that are technologically and/or spa-
tially seperated. The intertemporal spillover of knowledge transfers imperfec-
tly between firms that are spatially separated. \( \lambda_R < 1 \) is a scalar that describes the
proportion of knowledge that is available to a firm that is spatially separated
from the location of that knowledge. We weight knowledge associated with the
quality level of firms in a different location by the spatial parameter \( \lambda_R \). The
intertemporal spillover of knowledge also transfers imperfectly between sectors
and producers of different varieties. The same as we think of firms separated
by geographic space, we can also think of manufacturers of different varieties
being separated by technological space. We assume that the knowledge of a
firm’s own variety is more useful (closer in technological space) than other va-
rieties and knowledge of varieties in the firm’s own sector are more useful than
knowledge of other sectors. In this way firms in one’s own sector are separated
by some technological space from the firm’s own variety and firms in other sec-
tors are separated by a greater technological space. We assume knowledge from
all sources is additive. The knowledge spillover therefore has three weighted
components:

- the knowledge associated with the firm’s own variety \( i, j \)

\[ A_{i,j,t-1} \text{ or } A_{i,j,t-1} \]

• the weighted average knowledge associated with varieties in the firm’s own
sector \( i \) weighted by location

\[ \overline{A}_{i \forall (n_i + \bar{n}_i), t-1} = \frac{\sum_{k \in n_i} A_{i,k,t-1} + \lambda_R \sum_{k \in \bar{n}_i} A_{i,k,t-1}}{n_i + \bar{n}_i} \]

• and the weighted average knowledge associated with varieties in all sectors
weighted by location

\[ \overline{A}_{i \forall M, t-1} = \frac{\sum_{m \in M, k \in n_m} A_{m,k,t-1} + \lambda_R \sum_{\tilde{m} \in M, k \in \tilde{n}_{\tilde{m}}} A_{\tilde{m},k,t-1}}{\sum_{m=1}^{M} n_m + \sum_{\tilde{m}=1}^{\tilde{M}} \tilde{n}_{\tilde{m}}} \]

\[ = \frac{\sum_{i \in M} \overline{A}_{i \forall (n_i + \bar{n}_i), t-1}}{M} \]
We also weight each component of knowledge according to a related variety (Boschma and Frenken, 2009) approach where the relatedness of technology describes how useful the knowledge is to innovation in a firm’s own variety. We give knowledge of a firm’s own variety a weighting of one, knowledge of varieties within the firm’s own sector a weighting of $\lambda_V$ and knowledge of varieties in other sectors $\lambda_M$, such that $\lambda_V > \lambda_M \geq 0$. If the firm’s selected variety was previously produced in foreign it has the same spatial weighting as any foreign knowledge $\lambda_R$. Of course the relatedness of different varieties in the real world is not so simple. We could weight knowledge from every individual pair of varieties by some kind of proximity (Boschma, 2005) measure from which firms choose an optimal location. For simplification we assume greater technological relatedness for firms in the same sector than firms in different sectors, that weightings are constant for all varieties within a firm’s sector and constant for all varieties not in the firm’s sector. For a home region firm producing variety $i,j$ the overall index of technological opportunity is given by:

$$A_{i,j,t-1} = \max \left( A_{i,j,t-1}, \lambda_R A_{i,j,t-1} \right) + \lambda_V A_{i,i',t-1} + \lambda_M A_{i,i',t-1} \quad (10)$$

That is, the index of technological opportunity is the maximum knowledge associated with the quality of the firm’s own variety weighted by location, plus a weighted average of the knowledge associated with the quality levels of all other varieties weighted by location and technological relatedness. As a result firms may face a trade-off between the costs of innovation by locating in a cluster or an agglomeration. It is this trade-off which leads to the possibility of including clusters in an endogenous growth model and has interesting implications for growth policy.

If the variety has never been produced before the knowledge of a firm’s own quality level is replaced by an average quality level for their selected sector $i$. This maintains symmetry in each sector even when a new variety is invented (and another variety disappears due to the endogenous constant number of varieties in each sector). The index of technological opportunity is given by:

$$A_{i,j,t-1} = \frac{\sum_{k \in n_i} A_{i,k,t} \cdot \lambda_V}{n_i} + \lambda_R A_{i,i',t-1} + \lambda_M A_{i,i',t-1} \quad (11)$$

Analogous equations exist for foreign firms.

Following their fixed investment in period $t-1$, firms may produce any quantity of their product in period $t$ at a constant marginal (labor) cost of $w\beta$. All workers provide one unit of production per period. The traditional goods sector has 1:1 technology such that total production of traditional goods is $C_T + C_{\bar{T}} = 1 - \mu$. 

$$13$$
2.2 Short Run Equilibrium

The short run equilibrium describes the prices, wages, production and investment in innovation in each period. Equilibrium prices, wages and production follows from typical optimisation of Cobb-Douglas utility and within each manufacturing sector from Dixit-Stiglitz utility. The overall optimisation is requires some modification for multiple manufacturing sectors and firms optimising for their investment in quality improving innovations based on the consumer’s taste for quality and firms’ access to knowledge spillovers.

2.2.1 Prices, Wages and Production.

Consumers allocate expenditure across varieties to maximize utility subject to the budget constraint

\[ P_T C_T + \sum_{i \in M} P_i C_i \leq E + \tilde{E} \]  

(12)

where \( E + \tilde{E} \) describes world expenditure, \( P_T \) is the price of traditional goods, \( P_i \) is the local price index of each manufacturing sector \( i \) and \( C_i \) is the local consumption index of each sector \( i \) given by equations 3 or 4. By optimization and the nature of Cobb-Douglas preferences, \( 1 - \mu \) is the share of expenditure spent on traditional goods and \( \frac{\mu}{M} \) is the share spent on all varieties in each sector.

\[ P_T C_T = (1 - \mu) E + \tilde{E}, \quad P_i C_i = \mu \frac{\mu}{M} \left( E + \tilde{E} \right) \quad \forall i \in M \]  

(13)

Traditional Goods  While we are most interested in the knowledge sectors characterized by manufactured goods with increasing quality levels, the traditional goods sector is included for several reasons. In the real world workers in the traditional goods sector are not necessarily unskilled or immobile but these are common properties of the traditional sector in trade models. A portion of the workforce remains in all locations, even if there is full agglomeration, so that regions continue to trade. Some factors of production in the real world such as land, are immobile; while we don’t include land as a factor of production the immobility of unskilled workers is a proxy for immobile production factors like land. That is not to say the types of clustering effects we describe in high-tech sectors are not also occurring in lower technology or traditional sectors, we simply make these assumptions in order to model the general equilibrium. The important feature of the traditional goods sector in the general equilibrium model is that the factor of production is immobile and some portion of all consumption is immobile; “unskilled labor” (or “peasants”) is the commonly used term for this factor in trade models. We address the traditional goods sector briefly and otherwise the the remainder of the analysis focuses on the knowledge sectors of innovation and manufacturing.

The traditional goods sector is perfectly competitive and has constant returns to scale. Unskilled workers provide one unit of production per period and
Free trade goods ensures equal nominal prices and wages in the two regions. With full employment of $1 - \mu$ unskilled workers, 1:1 technology and normalising world expenditure $E + \tilde{E} = 1$, the traditional goods sector is the numeraire.

$$w_T(C_T + \tilde{C}_T) = w_T(1 - \mu) = P_T(C_T + \tilde{C}_T) = P_T(1 - \mu) \left( E + \tilde{E} \right)$$

$$w_T = \tilde{w}_T = P_T = \tilde{P}_T = E + \tilde{E} = 1$$

(14)

Manufacturing  Cobb-Douglas preferences between manufacturing sectors mean each variety competes only with its own sector via CES preferences. In the manufacturing sector we find the demand functions for home consumers of individual varieties. The demand function from all consumers for variety $i,j$ produced in the home region is

$$c_{i,j} = \frac{\mu (E + \tilde{E})}{M} A_{i,j}^{\sigma} \tilde{P}_{i,j}^{1-\sigma} P_{i}^{\sigma - 1}$$

(15)

where $P_i$ is an index for sector $i$ of the price and quality of all varieties in that sector such that:

$$P_i = \left[ \sum_{k=1}^{n_i + \tilde{n}_i} A_i^{\sigma - 1} p_i^{\sigma - 1} \right]^{1 - \sigma}$$

$$= \left[ n_i A_i^{\sigma - 1} p_i^{\sigma - 1} + \tilde{n}_i A_i^{\sigma - 1} \tilde{p}_i^{\sigma - 1} \right]^{1 - \sigma}$$

(16)

In the period prior to production, potential investors/firms choose whether to enter, and if they do enter, they select a sector, variety and a level of quality improvement. Since each niche monopoly is contestable by quality improvement, no firm can appropriate the intertemporal knowledge spillover and a fixed cost investment must be made each period in order to produce. We assume no two firms will choose to produce the same variety and we assume no economies of scope. There is no strategic interaction so production takes place under symmetric monopolistic competition with other varieties in the same sector. Firms do not compete with other sectors as varieties are only substitutes within the sector and not substitutes for varieties in other sectors. Since all varieties in the sector are symmetric, quality will be the same for each variety in each specific sector but quality levels do not have to be the same across sectors. Relative quality magnitudes between varieties within each sector describe the relative effect on consumer utility. However because firms compete only with their own sector relative quality magnitudes when compared across sectors instead describe the knowledge intensity of each sector as an input to innovation relative to the knowledge inputs from other sectors. We term this comparative technology “own sector knowledge intensity” and it is expressed by a relatively higher $A_i$ for sector $i$. That is, if sector $i$ has a higher own sector knowledge intensity, it means firms in sector $i$ source a higher share of knowledge from their own
sector compared to firms in other sectors who source a lower share from their own sectors. It is important for considering the role of inter-sectoral knowledge spillovers. Manufacturing firms invest in R&D, paid for by future sales revenues.

In period $t - 1$ each firm selects a quality improvement for production in period $t$ with its associated cost of innovation (equation 9) and its period $t$ price to maximise the monopolistically competitive profits discounted for time:

$$\max_{p_{i,j,t}, A_{i,j,t}} \left( \frac{(p_{i,j,t} - \beta)(c_{i,j,t} + \bar{c}_{i,j,t})}{1 + r_t} - F_{i,j,t-1} \right) \left( A_{i,j,t}, \bar{T}_{i,j,t-1} \right)$$

where $\beta$ is the marginal cost of producing one more unit of variety $i,j$ and is the per unit labor requirement ($w$ is the numeraire so is not included for simplification). $F_{i,j,t-1}$ is the number of knowledge workers required by the firm in the R&D sector to achieve a target quality level of $A_{i,j,t}$. Each firm maximises profit subject to the demand functions in each location for local and imported (exported by the firm) varieties. A home region manufacturer is subject to the demand function given by equation 15. As innovation occurs in the period prior to production firms discount future profit for time. We assume each firm takes expected price and quality setting behaviour of other firms as given and firms ignore the effects of their own pricing decisions on the price quality index. i.e. we treat $P_i$ and $\tilde{P}_i$ as fixed when differentiating by $p_i$. These assumptions are plausible with a sufficiently large number of firms in each sector. Firms compete using Bertrand competition although with a sufficiently large number of firms, the mode of competition (Bertrand or Cournot) has no effect. By differentiation we find the first order conditions:

$$c_{i,j,t} + (p_{i,j,t} - \beta) \frac{\partial c_{i,j,t}}{\partial p_{i,j,t}} = 0 \quad (18)$$

$$\left( \frac{(p_{i,j,t} - \beta)}{1 + r_t} \right) + \left( \frac{(p_{i,j,t} - \frac{1}{\tau} \beta)}{1 + r_t} \right) \frac{\partial c_{i,j,t}}{\partial A_{i,j,t}} - \frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}} = 0 \quad (19)$$

As is usual in Dixit-Stiglitz models we assume free entry and the number of varieties in each sector is determined endogenously by the elasticity of substitution and the size of each region. With free entry into the R&D sector firms make zero profits as all margin is invested in innovation. Profits are zero in equilibrium because if profits were positive, the marginal skilled worker could shift to the R&D sector and produce greater quality improvements or an additional variety. We have the free entry condition:

$$\left( \frac{(p_{i,j,t} - \beta)}{1 + r_t} \right) c_{i,j,t} + \left( \frac{(p_{i,j,t} - \frac{1}{\tau} \beta)}{1 + r_t} \right) \bar{c}_{i,j,t} = F_{i,j,t-1} \left( A_{i,j,t}, \bar{T}_{i,j,t-1} \right)$$

From the first two first order conditions we find firms set prices to a constant mark-up over marginal cost:
where we have included wages (which are the numeraire) for illustrative purposes only. With zero transport costs, exported varieties have the same prices as domestic varieties. Firms have the same margin per unit of domestic or exported production.

### 2.2.2 R&D Investment and Quality Improvement

Dividing the third first order condition (equation 20) by the free entry condition (equation 21) and rearranging, firms select a quality improvement where the elasticity of research cost with respect to quality is equal to the elasticity of demand with respect to quality.

\[
\frac{\varepsilon c_{A_{i,j,t}} + \frac{1}{\sigma} \varepsilon c_{A_{i,j,t}}}{\sigma - 1} = \frac{\varepsilon F_{A_{i,j,t}}}{\eta A_{i,j,t-1}} \tag{23}
\]

By rearrangement we have

\[
\frac{\sigma - 1}{\eta} = \frac{A_{i,j,t}}{A_{i,j,t-1}} \tag{24}
\]

which describes a willingness for firms to invest in quality improvement. By substitution into equation 9 the cost of innovation or willingness to invest per period per firm is:

\[
F_H = \gamma e^{\frac{A_{i,j,t}}{\eta}} = \gamma e^{\sigma - 1} \tag{25}
\]

Rearranging further, firms select a quality target of:

\[
A_{i,j,t} = \frac{\sigma - 1}{\eta} \left[ \max \left( A_{i,j,t-1}, \lambda_R A_{i,j,t-1} \right) + \lambda_V \overline{A}_{i,j,t-1} + \lambda_M \overline{A}_{i,j,t-1} \right] \tag{26}
\]

This is a quality improvement multiplier of:

\[
\frac{A_{i,j,t}}{\max \left( A_{i,j,t-1}, \lambda_R A_{i,j,t-1} \right)} = \frac{\sigma - 1}{\eta} \left[ 1 + \frac{\lambda_V \overline{A}_{i,j,t-1} + \lambda_M \overline{A}_{i,j,t-1}}{\max \left( A_{i,j,t-1}, \lambda_R A_{i,j,t-1} \right)} \right] > 1 \tag{27}
\]

We assume this multiplier is greater than one such that there are always quality improvements in equilibrium. Intuitively equation 26 describes two parts, it is the quality improvement due to the firms willingness to pay for R&D plus the variety specific knowledge spillover. Since the sector is symmetric all firms in each sector are the same size and have equal quality improvements in the steady state.
2.2.3 Labour Market Clearing and Endogenous Variety

Labor market clearing requires that the total labour used in home region manufacturing ($L_M$) and R&D ($L_R$) be equal to the total supply of home region skilled workers ($L_K$). The skilled labor used in manufacturing in the home region is the worldwide expenditure on manufactured goods produced in the home region divided by the price per unit multiplied by its marginal cost. Since we have a model with homogenous skilled workers there is no need to distinguish between varieties of goods in each industry $i$ or to distinguish between industries. In the perfect symmetric equilibrium the number of skilled workers used in goods production by all home manufacturers is:

$$L_M = \frac{\mu \left( sE + (1 - \tilde{s}) \tilde{E} \right)}{p} \beta = \frac{\sigma - 1}{\sigma} \mu \left( sE + (1 - \tilde{s}) \tilde{E} \right)$$

where $s = \frac{\sum_{i=1}^{M} c_i}{\sum_{i=1}^{M} c_i + \sum_{i=1}^{M} \tilde{c}_i}$ is the domestic market share of manufacturing by home region firms and $(1 - \tilde{s})$ is their export market share. The labor used in research is equal to the number of firms in the next period multiplied by the investment in research labour by each individual firm:

$$L_R, t = \gamma e^{\sigma - 1} \sum_{i=1}^{M} n_{i, t+1}$$

Analogous equations exist for the foreign region.

The reward for investing in R&D to develop a quality improvement is the operating profit from the following period. With constant mark-up over marginal cost the operating profit $\pi$ is the value of sales shared equally between firms, divided by $\sigma$: $\pi = \frac{1}{\sum_{i=1}^{M} n_i} \mu \left( s_iE_i + (1 - s_i) \tilde{E}_i \right)$. By the free entry condition (equation 21) the amount each firm spends on innovation is their present value share of profit. The free entry relation can be rearranged to:

$$\pi_t = \frac{\mu \left( s_tE_t + (1 - s_t) \tilde{E}_t \right)}{\sigma \sum_{i=1}^{M} n_{i, t}} \frac{1}{1 + r} = \gamma e^{\sigma - 1} = F_{t-1}$$

Substituting this expression (advanced one period) into equation 30, we have:

$$L_K, t = \frac{(\sigma - 1)}{\sigma} \mu \left( s_tE_t + (1 - s_t) \tilde{E}_t \right) + \frac{\mu \left( s_{t+1}E_{t+1} + (1 - s_{t+1}) \tilde{E}_{t+1} \right)}{\sigma (1 + r)}$$
Rearranging the Euler equation given in equation 5, \( \frac{E_{t+1} + \tilde{E}_{t+1}}{1 + \rho} = \left( E_t + \tilde{E}_t \right) \alpha \), we can solve for the value of consumer expenditure on manufactured goods as a function of the parameters:

\[
\mu \left( s_t E_t + (1 - s_t) \tilde{E}_t \right) = \frac{\sigma L_{K,t}}{(\sigma - 1) + \alpha} \tag{33}
\]

Equilibrium requires the economy move to a steady state level of consumer expenditure with a constant interest rate \( 1 + r = 1 + \rho = \frac{1}{\alpha} \). Substituting (32) into the modified free entry relation (30) and rearranging allows us to determine the total number of firms per region across all industries as a function of the parameters.

\[
\sum_{i=1}^{M} n_i = \frac{L_K \alpha}{[(\sigma - 1) + \alpha] \gamma e^{\sigma - 1}} \tag{34}
\]

An analogous function exists for foreign firms. Alternatively we can use similar rearrangements to find the total number of firms in each industry:

\[
n_i + n_{\tilde{i}} = \frac{\left( L_K + \tilde{L}_K \right) \alpha}{M [(\sigma - 1) + \alpha] \gamma e^{\sigma - 1}} \tag{35}
\]

### 2.3 Steady State Allocation of Manufacturing

The steady state is defined by a constant number of manufactured varieties, constant growth in the quality of varieties in each sector, constant prices and quantities of production and constant division between regions. Here we study the division of firms between regions. We consider any distribution of firms in sector \( i \) and study the incentives for a firm to switch location from the home to the foreign region.

With zero transport costs, the value of sales for a home region firm \( (V) \) is equal to the value of sales if the firm were to switch to a foreign region \( (\tilde{V}) \). Their equally divided share of world consumer expenditure on sector \( i \):

\[
V = \frac{\mu \left( E + \tilde{E} \right)}{M (n_i + n_{\tilde{i}})} = \tilde{V} \tag{36}
\]

By the value of sales alone firm location would be determined as in Krugman (1991) such that with zero transport costs firms are indifferent between locations. Firms could be located in either region and any distribution of firms is possible. However transport costs aren’t the only influence on firm location. In the model here firms consider cost differences in innovation which includes the location and proximity of firms in their own sector and other sectors. We therefore also consider the fixed costs of innovation for the home region firm \( (F) \) and the fixed costs of innovation for the foreign region firm \( (\tilde{F}) \).
2.3.1 Contestability, Quality Improvement and Market Entry Criteria

For a firm to enter its market in each period the firm must achieve the same level of quality improvement to be equal with other firms in their sector. If the firm achieves a lower quality improvement contestability and the free entry condition would mean an entrant could innovate by a greater amount and replace the firm within its niche (variety). The firm would not be able to produce at all because it could not achieve a patent by having the greatest quality improvement for its niche. Thus, to consider the possibility of a firm switching location we treat it as possible that a firm might choose to locate in a foreign region even if $\tilde{F} > F = \gamma e^{\sigma r}$ (or a home region even if $F > \tilde{F} = \gamma e^{\sigma r}$). Where a firm is in a less than optimal R&D location, the firm must make additional investment in R&D to achieve entry to the market. That is, they will have a higher cost of innovation.

With symmetric firms in each sector the knowledge input to innovation for a home region firm is

$$\tilde{A}_{i,t-1} = A_{i,t-1} \left[ 1 + \lambda V \left( \frac{n_i + \lambda R n_i}{n_i + n_i^*} \right) \right] + \lambda M \left( \frac{\sum_{m=1}^{M} n_m A_{m,t-1} + \lambda R \sum_{\tilde{m}=1}^{\tilde{M}} n_{\tilde{m}} \tilde{A}_{\tilde{m},t-1}}{\sum_{m=1}^{M} n_m + \sum_{\tilde{m}=1}^{\tilde{M}} n_{\tilde{m}}} \right) \tag{37}$$

Its willingness to invest in innovation is given by equation 25 and its targeted quality level multiplier is given by equation 27. For a home region firm that switches location, the cost of quality improvement is the same as in equation 25 (with notation $\tilde{F}_H$) but with the knowledge input to innovation adjusted by the new location of the firm:

$$\tilde{A}_{i,t-1} = A_{i,t-1} \left[ 1 + \lambda V \left( \frac{\lambda R n_i + n_i^*}{n_i + n_i^*} \right) \right] + \lambda M \left( \frac{\lambda R \sum_{m=1}^{M} n_m A_{m,t-1} + \sum_{\tilde{m}=1}^{\tilde{M}} n_{\tilde{m}} \tilde{A}_{\tilde{m},t-1}}{\sum_{m=1}^{M} n_m + \sum_{\tilde{m}=1}^{\tilde{M}} n_{\tilde{m}}} \right) \tag{38}$$

A foreign firm selects a quality improvement for production in period $t$ with its associated cost of innovation and its period $t$ price to maximise the monopolistically competitive profits discounted for time. Foreign firms also prefer to select a quality improvement where the elasticity of research cost with respect to quality is equal to the elasticity of demand with respect to quality. Elasticities are the same in either region so equation 24 for the foreign firm is:

$$\frac{\sigma - 1}{\eta} = \frac{\tilde{A}_{i,j,t}}{A_{i,j,t-1}} \tag{39}$$
The foreign firm has the same willingness to invest in innovation \( \bar{F}_{i,j,t} = \gamma e^{\sigma - 1} \) as the home region firm. Rearranging 39, the desired level of quality improvement is given by \( A_{i,t} = \frac{\sigma - 1}{\eta} \bar{A}_{i,t-1} \) and \( \bar{A}_{i,t} = \frac{\sigma - 1}{\eta} \bar{A}_{i,t-1} \).

In assessing the costs and benefits of each alternative location we take the approach that contestability and the entry criteria requires the firm achieve the greatest quality improvement available from the alternative location choices. That is, \( A_{i,t} = \max \left( \frac{\sigma - 1}{\eta} \bar{A}_{i,t-1}, \frac{\sigma - 1}{\eta} \bar{A}_{i,t-1} \right) \) even if \( F_{i,j,t} > \gamma e^{\sigma - 1} \). In the model here the only variation between locations is the cost of innovation so a firm would never choose a less than optimal location for R&D. But in reality there are many more factors to consider for firm location such that a firm may choose a less optimal location for other advantages. In the real world firms would also consider factor prices, trade costs and the value of sales in each location in addition to these innovation cost factors considered here. These additional factors complicate the model so are left aside for now. Greater value of sales or lower factor prices could justify a firm choosing a location that is less than optimal for R&D (or a firm choosing a more optimal location for R&D despite higher factor prices or a lower value of sales). The firm may be willing to do this if there is additional value available by choosing that location. The purpose of this approach is that the model is comparable with other two region models and allows additional factors of firm location to be added (such as transport costs).

### 2.3.2 The Requirement for a Steady State

The steady state is usually defined as an unchanging growth rate in the number of manufactured varieties, its regional division, as well as the prices and quantities defined by short run equilibrium above. Migration of knowledge workers due to spatial inequality of real wages and the switching of firms due to differences in knowledge spillovers leads to the long run equilibrium. We define the steady state similarly, but only across at least two periods rather than indefinitely. This as a steady state, because between (at least) two periods, firms and workers have no incentive to switch region and therefore the economy is “steady”. This is different to the traditional definition of a steady state because within the model here there emerges medium term steady states, which were not possible in earlier two region growth models.

Consider a cluster of firms in the home region. A home region firm in that cluster will only switch if the firm can achieve greater return on investment in the new location.

\[
\frac{\tilde{V}}{wF} > \frac{V}{wF}, \quad \tilde{F} < F
\]  

In the initial model with zero transport costs and mobile skilled workers; wages, prices and value are equal in both locations in the steady state. Rearranging equation 40 a firm will choose the location where the cost of innovation is the lowest (knowledge spillovers are greatest). The requirement for a firm to locate in the home region in the steady state simplifies to:

\[
\frac{\tilde{V}}{wF} > \frac{V}{wF}, \quad \tilde{F} < F
\]
\[ \tilde{F} \geq F, \quad \forall j, i \]  

(41)

All firms in the same sector have the same cost of innovation. If a single firm

switches, the cost of innovation in the foreign region for other firms in the same

sector will decrease. Therefore if equation 40 does not hold true for one firm

in sector \( i \) such that the firm switches location, it will not hold true for other

firms in sector \( i \) (even more so after the first firm switches) such that the entire

sector will also switch location. As a result, each sector will remain clustered

in its historical location, until equation 40 no longer holds. We therefore only

have to consider steady states where each sector is clustered in single locations.

Knowledge spillovers increase with sector concentration and with manufacturing

concentration. These two factors determine firm location. Firstly firms

prefer locations with a greater share of their own sector. We term this additional

force for sectoral concentration, the clustering effect. But firms must balance

this attraction with a preference to locate where there are more firms overall

because greater concentration of all manufacturers also increases knowledge

spillovers. We term the alternative force for firm location, the agglomeration

effect. Depending on the distribution of each sector these forces may be in the

same direction but could potentially be in the opposite directions if sectors are

in peripheral clusters. In this way clustered sectors may sustainably produce in

a peripheral location.

As described above, the sectoral quality level is set by the highest level of

quality from either region that is available for the fixed cost of \( \gamma \sigma^{-1} \). Assume

this quality level is obtained in the home region such that \( \tilde{A}_{i,j,t} < A_{i,j,t} \) and

\( F = \gamma \sigma^{-1} \). The cost of achieving the quality level \( A_{i,j,t} \) for a firm switching to

the foreign location is:

\[ \tilde{F} = \gamma e^{\frac{n_{A_i,t}}{\tilde{A}_{i,t}}} \]  

(42)

Firms select a quality improvement target determined in the home location

given by equation 26. The intertemporal spillover of the firm’s own knowledge

diminishes by \( 1 - \lambda_R \) when the firm switches. Substituting the knowledge input

(11), modified for the foreign region, and the targeted quality level (26) into

equation 42:

\[ \tilde{F} = \gamma e^{\frac{A_{i,t} - 1 + \lambda_V \tilde{A}_{i,t}}{\tilde{A}_{i,t}}} + \lambda M \tilde{A}_{i,t}} \]  

(43)

The difference between entry costs in the home region \( F \) and entry costs in

the foreign region \( \tilde{F} \) is the exponent in \( \tilde{F} \), is multiplied by the ratio of knowledge

from all firms in either location, where the weightings depend on the locations

of firms relative to each alternative region. The relative magnitudes of \( F \) and

\( \tilde{F} \) depend upon the distribution of firms within sector \( i \) and on the distribution

of all firms. If foreign knowledge spillovers are lower there will be a greater cost

of innovation in the foreign region as given by equation 43. Substituting (43)
and \( F_H = \gamma e^{\sigma - 1} \) into equation 41 and rearranging we see that in equilibrium the firm chooses the location where knowledge spillovers are greatest:

\[
A_{i,t-1} + \lambda_V \overline{A}_{i\forall(n_i + n_j),t-1} + \lambda_M \overline{A}_{\forall M,t-1} \geq \lambda_R A_{i,t-1} + \lambda_V \overline{A}_{i\forall(n_i + n_j),t-1} + \lambda_M \overline{A}_{\forall M,t-1} \tag{44}
\]

By symmetry and clustering of each sector, \( A_{i,t-1} = \overline{A}_{i\forall(n_i + n_j),t-1} \) and \( \lambda_R A_{i,t-1} = \overline{A}_{i\forall(n_i + n_j),t-1} \). Rearranging further

\[
A_{i,t-1} \geq \frac{\lambda_M}{(1 - \lambda_R + \lambda_V - \lambda_V \lambda_R)} \left( \overline{A}_{\forall M,t-1} - \overline{A}_{i\forall M,t-1} \right) \tag{45}
\]

If sector \( i \) has a quality parameter greater than this threshold level than it is possible for sector \( i \) to be clustered in the home region in the current period, even if the home region is not the location of other sectors. If all other sectors are agglomerated in foreign, this increases the threshold for the quality parameter in sector \( i \). We define this as a steady state because no single firm would switch if it had the opportunity, there is no change in labour endowments in each region, all firms in each industry grow at the same rate and will continue to grow at the same rate in future periods if the threshold continues to be met.

However to be a steady state in the long run, this quality parameter threshold must be met for longer than one period. Over time, quality in the agglomerated sectors in the foreign region may grow faster than quality in the peripheral cluster because the agglomeration results in greater knowledge spillovers for those sectors. Therefore the level of inter-sectoral spillovers, if the firm were to switch to the foreign region \( \overline{A}_{i\forall M,t-1} \), will also grow faster than the level of inter-sectoral spillovers in the home region \( \overline{A}_{i\forall M,t-1} \). As a result the peripheral cluster may not be sustainable indefinitely. We therefore define these steady states as medium-term.

### 2.3.3 Requirement for a Dynamic Steady State

We define a dynamic steady state as a steady state which is sustainable indefinitely. To be a dynamic steady state the threshold given by equation 45 must be met for all time periods. We can intuitively see two obvious dynamic steady states. The core-periphery outcome where all sectors cluster in a single region is a dynamic steady state where all firms benefit from co-locating. No firm will switch location and there is no technology intensity catch-up in the periphery to induce a firm or sector to switch location. Alternatively the equal distribution outcome where half the sectors are clustered in each region is a steady state if there is also an equal distribution of technology intensities and workers. Each location will have equal growth in the quality levels of comparable technology intensive industries, so there will be no incentive for firms or workers to switch location.

It is possible for a peripheral cluster equilibrium to also be a dynamic steady state if the increases in quality levels in the peripheral cluster are greater than
the increase in the threshold. Quality change over time is found by rearranging equation 26. The dot above the variable denotes the difference with respect to time (equivalent of differentiation with discrete time)

\[ \dot{A}_{i,t} = A_{i,t} - A_{i,t-1} = \frac{\sigma - 1}{\eta} \left[ A_{i,t-1} + \lambda_V A_{i,t-1} + \lambda_M \bar{A}_{i\forall M,t-1} \right] - A_{i,t-1} \quad (46) \]

Differencing the threshold (45), we have an additional threshold for a long-run steady state:

\[ \dot{A}_{i,t} \geq \frac{\lambda_M}{(1 - \lambda_R + \lambda_V - \lambda_V \lambda_R)} \left( \hat{A}_{\forall M,t} - \bar{A}_{\forall M,t} \right) \quad (47) \]

Note spillovers from all other firms are simply a weighted average of the knowledge intensity of all other sectors and note that each sector is symmetric. Examining \( \left( \hat{A}_{\forall M,t} - \bar{A}_{\forall M,t} \right) \) more closely we can rewrite equation 47 as:

\[ \dot{A}_{i,t} \geq \frac{\lambda_M}{M (1 - \lambda_R + \lambda_V - \lambda_V \lambda_R)} \left[ \sum_{j \in M} \left( \dot{A}_{j,t} (\lambda_R - 1) + \dot{A}_{\tilde{j},t} (1 - \lambda_R) \right) \right] \quad (48) \]

Since each industry is clustered only in one location either \( \dot{A}_{j,t} \) or \( \dot{A}_{\tilde{j},t} \) applies to the spillovers from each of the other industries (the other \( A \)'s are zero because the industry doesn’t exist in that location). Threshold (45) is met in all time periods for industry \( i \) if threshold (48) is met.

If growth in quality is greater than this second threshold (48), then growth in the cluster will increase the spillovers to other industries. This will eventually be strong enough to attract firms and sectors from the agglomerated location. These would start initially with those sectors that have the lowest quality parameters. These are the sectors with the lowest own sector knowledge intensity that benefit the most from spillovers from other sectors. If quality in all sectors in the agglomerated location was sufficiently low, then all manufacturing would agglomerate in the home region. However if some sectors in the agglomerated location have a high enough quality parameter, the dynamic steady state would be reached where the quality increases for the marginal sector in the peripheral cluster exactly equal the increase in the threshold for their industry. The marginal sector is the sector with the lowest own sector technology intensity in the location with the smaller share of manufacturing. A Dynamic Steady State requires the threshold (given by equation 48) be met in the marginal sector. In addition the Dynamic Steady state requires the inter-sectoral spillovers from either region are growing by the same amount. That is:

\[ \dot{A}_{\forall M,t} - \bar{A}_{\forall M,t} = 0 \quad (49) \]

Lastly consider the likelihood of this alternative equilibria where a peripheral location can attract industry from the core, and become the new agglomerated
The parameter A is probably normally distributed across all sectors from both regions. Since the final equilibria is largely determined by the starting parameters, now consider all the possible starting shares of industry, giving an equal probability to each. The most common equilibria will be where sectors with a low own sector technology intensity switch quickly to one region along with workers who follow the greater demand for skilled labour. This region becomes the core, and has it’s initial endowment of high technology intensity sectors and all low technology intensity sectors (as these sectors are more mobile). The periphery will maintain it’s initial endowment of high technology sectors for some time, but eventually all industry will agglomerate in the core as in the medium term steady state with peripheral clusters described in section 2.3.2. It is not very probable that a single region is initially endowed with a large share of low technology intensity industries and a larger share of all industry, while the other region is endowed with a high technology intensity industries, such that quality improvements in an initial periphery eventually attracts firms in the core to switch regions. Given this low probability we focus on the model with peripheral clusters in the medium term steady state and the implications for growth policy.

### 2.3.4 Consistency with Existing Literature

The model here is consistent with Duranton (2007) and Brezis and Krugman (1997) but with the additional richness of explaining an endogenous switching of industries between peripheral and core locations. Both of these models explain the rise and fall of locations through the stochastic emergence of new technologies in new locations but fail to include any endogeneity in why a location might eventually become a less than optimal choice for innovation activities. Here we show how greater technical externalities in agglomerated locations lead to greater change in technology and the eventual shifting of sectors from peripheral clusters to the agglomeration. Alternatively the model here shows how a new agglomeration can attract other industry, through the existence of a cluster that has a very high level of own sector technology intensity. While this alternative equilibria is unlikely, the possibility has implications for peripheral locations to target R&D incentives towards industries with a high level of own industry technology intensity.

But does this mean knowledge sectors typically cannot remain in the periphery indefinitely? If there were a stochastic addition to the model, such as a probability of R&D investment also developing a new variety or technology in an alternative field, this could lead to the emergence of new peripheral clusters and the constant shifting of peripheral clusters between peripheral locations and agglomerated locations. Considering the model here alongside Duranton (2007) or Brezis and Krugman (1997) results in a model with partially endogenous switching of industry from peripheral to core locations and partially stochastic churning of industry between locations. In addition, new industries are more likely to emerge in already agglomerated locations, but peripheral clusters will remain part of the economic landscape, producing quality improvements, but
at a lower frequency than core locations. Of the industries which emerge in the
periphery, only the sectors with a high level of own sector technology intensity
can remain sustainable in the medium term.

3 Growth

Quality of each variety grows at a rate given by rearranging equation 27:

\[
g_{A,i} = \frac{A_{i,j,t}}{\max(A_{i,j,t-1}, \lambda_R A_{i,j,t-1})} - 1
\]

\[
= \frac{\sigma - 1}{\eta} \left[ 1 + \lambda_V + \frac{\lambda_M \overline{A}_{i \forall M,t-1}}{\max(A_{i,j,t-1}, \lambda_R A_{i,j,t-1})} \right] - 1
\]

 Peripheral clusters are possible and sustainable in the medium and long run
provided the cluster meets the threshold of own industry technology intensity.

3.1 Simulations

We use simulations to demonstrate growth in quality levels and firm location
where own industry technology intensity for each cluster meets the requirement
for a medium term steady state. There are four possible scenarios:

1. Equal Distribution
2. Core-Periphery
3. Medium Term Peripheral Cluster
4. Long Term Peripheral Cluster

3.1.1 Parameters

We calibrate the model based on parameters used in other simulations. Krug-
man (1991) uses the parameters \( \sigma = 4 \) and \( \mu = 0.3 \). In a product variety growth
model Baldwin and Forslid (2000) use the parameters \( \sigma = 5, \mu = 1/4 \) and \( \alpha = 1/2 \)
(this implies an annual discount rate of aprox 7\% when periods are 10 years).
Since we have multiple sectors where firms only compete with their own sector
the elasticity of substitution within each sector in our model should be consid-
erably higher while the elasticity of substitution between varieties in different
industries is zero. To determine appropriate parameters for the multi-industry
model we maintain an average elasticity of subsitution the same as Baldwin
and Forslid (2000). With M industries and \( n_i + \tilde{n}_i = n \) firms in each industry
there are \( \frac{Mn(Mn-1)}{2} \) pairs of products. Only varieties within the same industry
are substitutes such that there are \( \frac{Mn(n-1)}{2} \) pairs with a non zero elasticity of

26
substitution. Following Baldwin and Forslid (2000) we set the average elasticity
\[ \tilde{\sigma} = \frac{M_n(n-1)\sigma}{M_n(M_n-1)} = 5 \]
such that we have an elasticity of \( \sigma = \tilde{\sigma} \frac{M_n-1}{n-1} \) for each
industry. If we set the number of firms per industry to 10 and the number of
industries to 10, this suggests an elasticity of \( \sigma = 55 \). We view 100 varieties
across 10 sectors as sufficient to demonstrate growth and location in a peripheral
cluster. Substituting into equation 35 along with \( \alpha = \frac{1}{4} \), \( M = 10 \), \( n_i + n_{\tilde{i}} = 10 \)
and \( (L_K + \tilde{L}_K) = \mu = \frac{1}{4} \) we find \( \gamma = \frac{(L_K + \tilde{L}_K)^{\alpha}}{M(n_i+n_{\tilde{i}})(\sigma-1)+\alpha e^{-(\sigma-1)}} = \frac{0.125}{5^{4/5}e^{\sigma}} \). The
last parameter to calculate is \( \eta \). As this is a parameter not used in typical
simulation models we calibrate \( \eta \) during simulations. We set initial values for
\( A_i \) equal to an average of one, normally distributed across the 10 sectors with
a standard deviation of 0.25. Knowledge spillover parameters will be varied to
show the effect on growth. Initially we start the simulations with the parameters
\( \lambda_R = 0.9 \), \( \lambda_V = 0.5 \) and \( \lambda_M = 0.1 \). Table 1 describes all the parameters used
here and a comparison to parameters used in other simulations of two region
models:

### 3.2 Simulations of Steady States

In this section we simulate each of the medium term steady states. Simulations
start with an initial endowment of firms, workers and technology levels in each
location, leading to a medium term steady state and eventually to a long-run
steady state. We estimate the rate of growth in quality levels and per capita
consumption in each location.

#### 3.2.1 Equal Distribution

In this simulation, each region has five sectors (50 firms) where each region has
the same initial distribution of technology intensities between sectors. A normal
distribution across 5 sectors implies
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<tr>
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<td>$M_a$</td>
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<tr>
<td>$\gamma$</td>
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<td>$\frac{0.125}{\text{max}^{55}}$ (to calibrate)</td>
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<tr>
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<td>$\lambda_M$ g</td>
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<td>0.1 (range 0-$\lambda_V$)</td>
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*aThese are not parameters for calibration, but the number of sectors and firms implied by the other parameters.

*bThese are not parameters for simulation, but the number of sectors and firms implied by the other parameters.

cThese are starting values only

dThese are starting values only

eThese are parameters for the initial simulation but in section 3.3 onwards these parameters will range across all possible values

fThese are parameters for the initial simulation but in section 3.3 onwards these parameters will range across all possible values

gThese are parameters for the initial simulation but in section 3.3 onwards these parameters will range across all possible values

Table 1: Simulation Parameters
3.2.2 Core-Periphery
3.2.3 Medium Term Peripheral Cluster
3.2.4 Long Term Peripheral Cluster
3.3 The Impact of Knowledge Spillovers upon Economic Growth
3.4 Agglomeration, Industrial Clusters and Economic Growth
3.5 Policy Implications for Economic Growth

4 Balancing Multiple Spatial Factors
4.1 Changes to Model Specification
4.1.1 Modified Entry Criteria
4.2 Short Run Equilibrium
4.3 Steady State Allocation of Industry
4.3.1 Modified Requirement for a Steady State
4.3.2 The Long-run Steady State
4.3.3 Interior Solutions
4.4 Simulations
Repeat of 3.2 above with additional interior industry solution.

5 Summary and Conclusion

References


